STRESS IN BEAM

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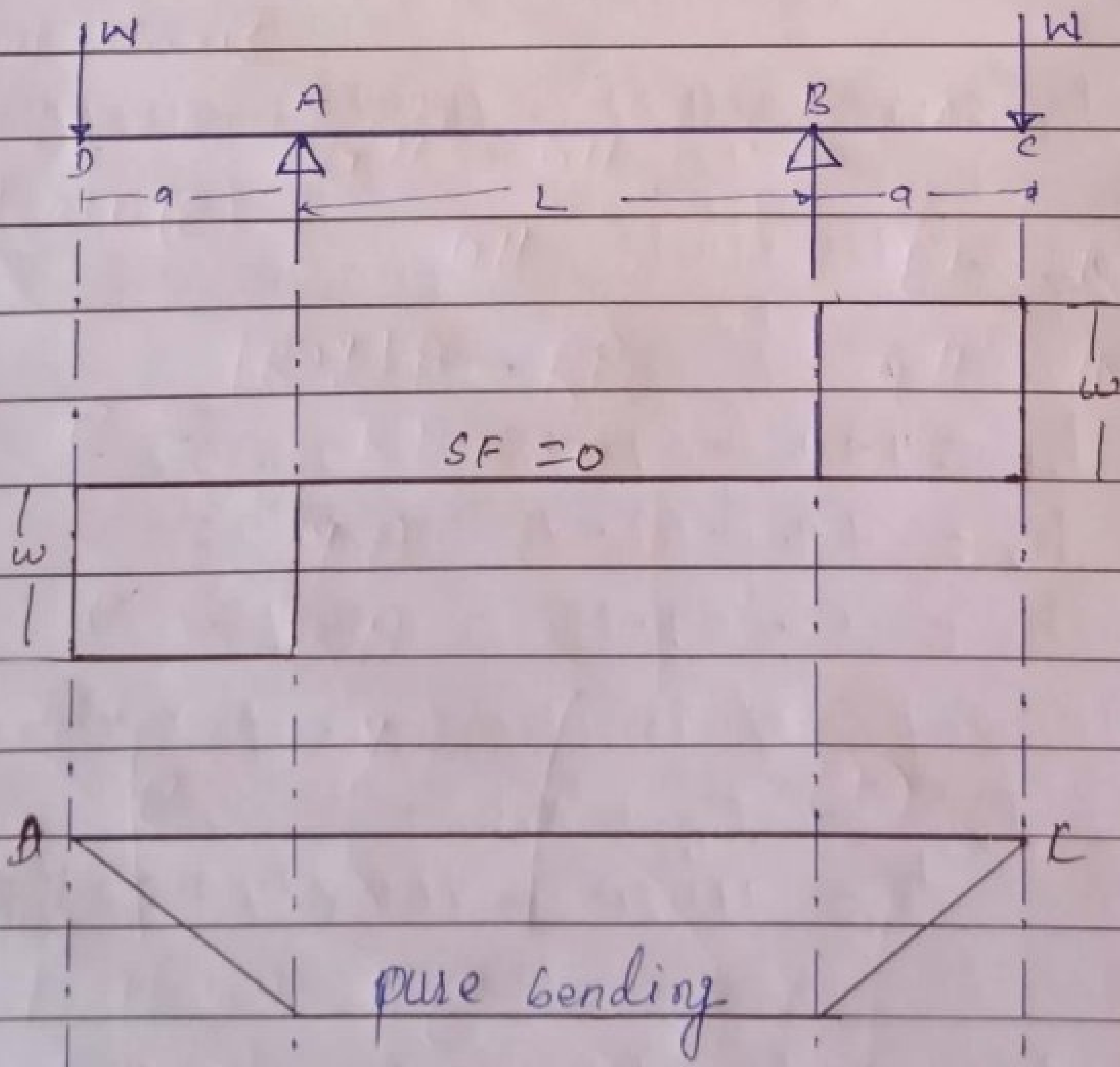
Stress
- Normal stress
  - ~~primary stress~~
  - Direct Normal stress
  - Bending stress
- Shear stress
  - primary shear stress
  - Torsional shear stress.

✡ PURE BENDING / SIMPLE BENDING:-

A beam is said to be under pure Bending if it is subjected to equal and opposite couple in a longitudinal plane in such a way that the magnitude of bending moment remains Constant and shear force will be zero.



⇒ if shear force at any section of beam is zero, while bending, then it is known as pure bending.

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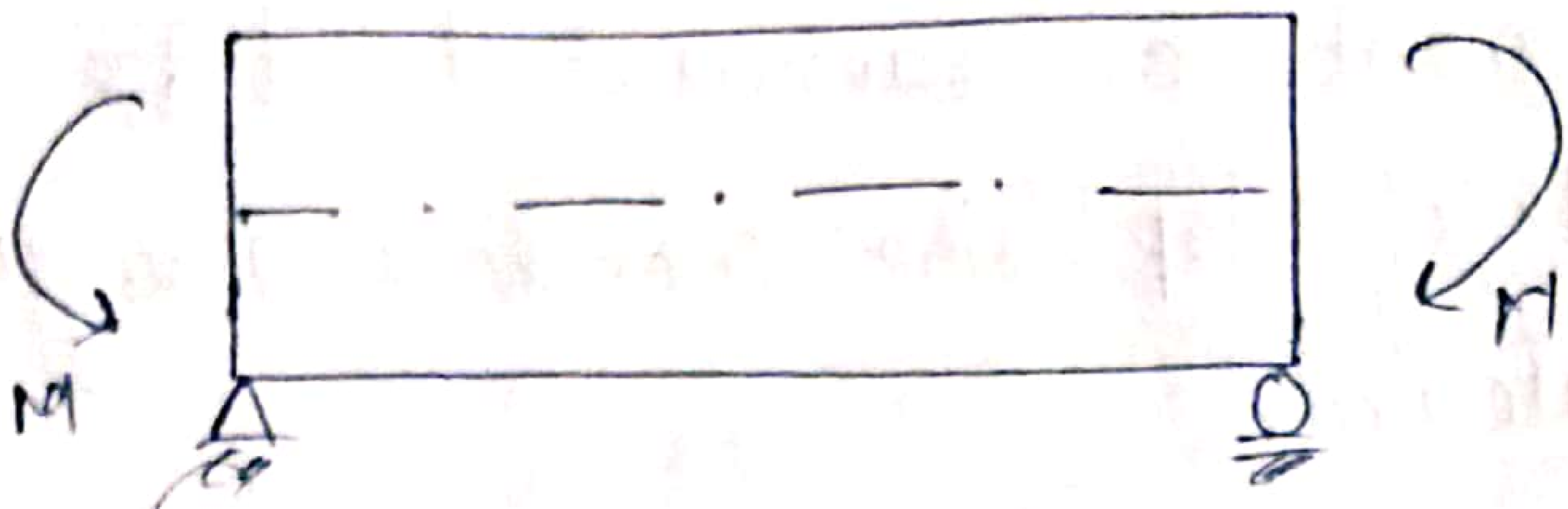
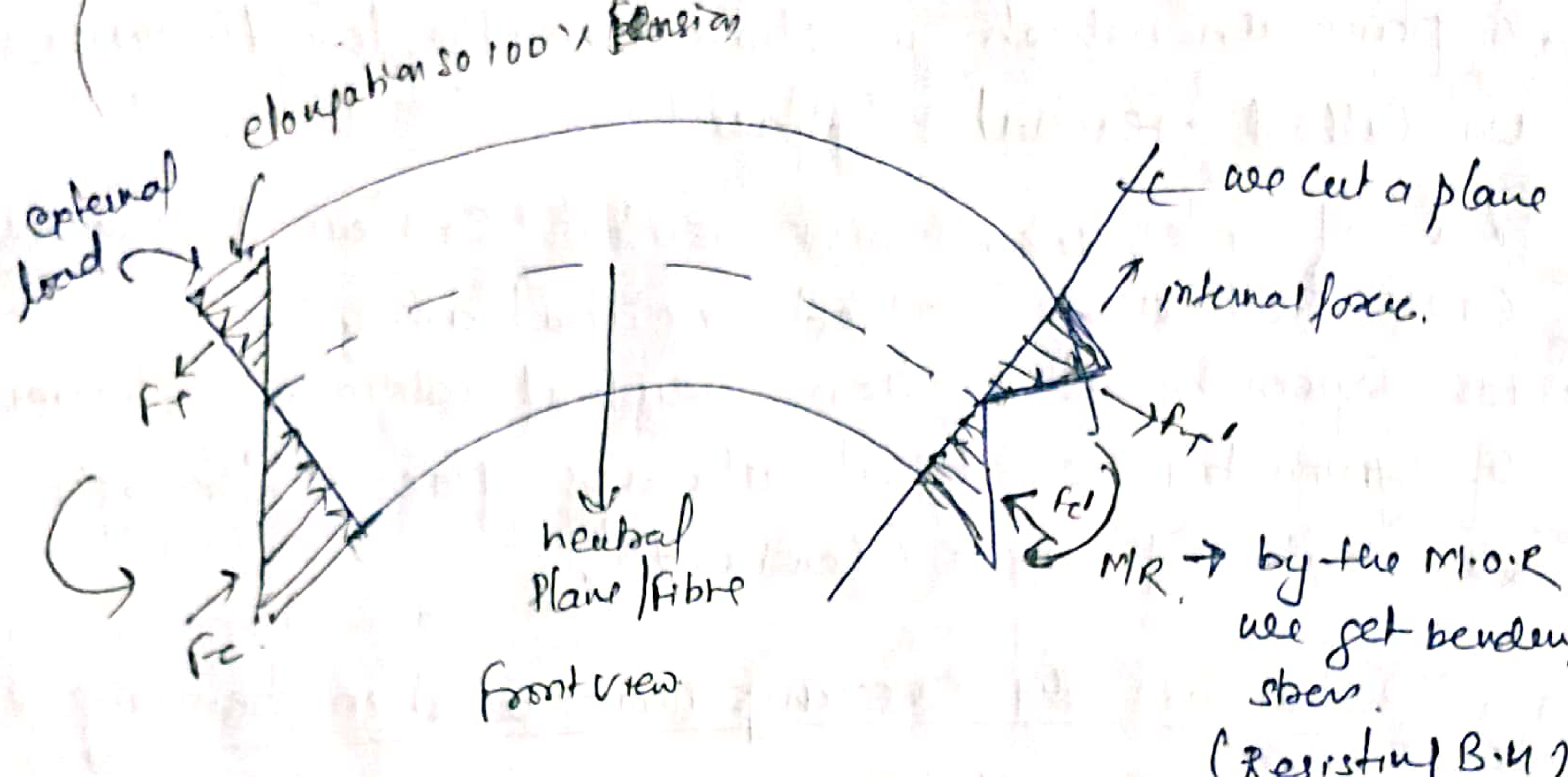
→ Bending stress / Flexural stresses in Beam :

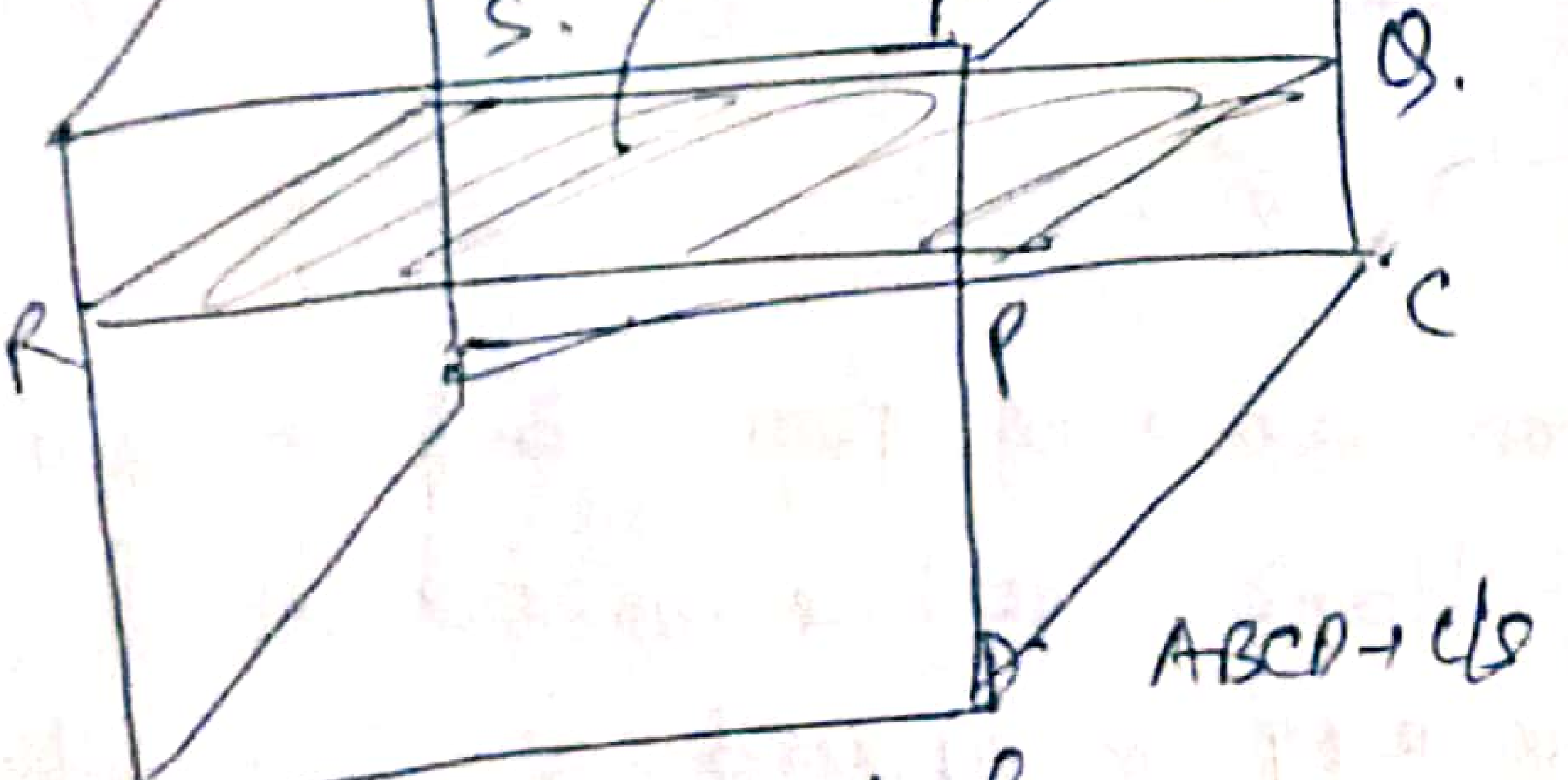


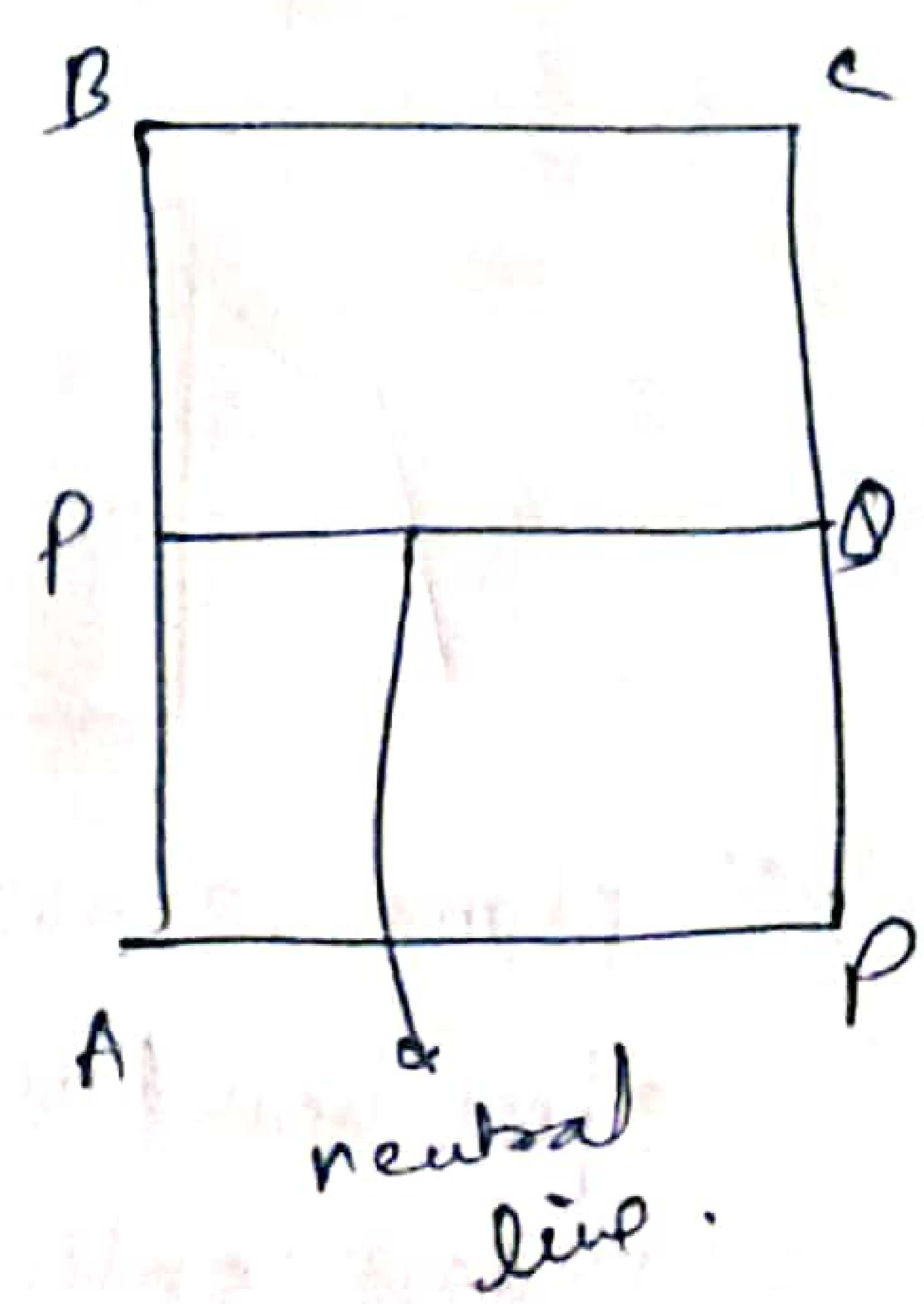


front view

Fc' & Ft' are internal force.

& $F_T' = F_T$ & $F_C' = F_C$.





PQRS → neutral plane
PQ → neutral line

A beam having an arbitrary c/s when subjected to transverse load or moment, it will bend. By applying the moment or transverse load As shown in fig, length of top fibre increases & length of bottom fibre decreases.

A plane in which no change in the length occurs is called neutral surface.

Line of intersection b/w neutral surface & transverse cross-section is called neutral axis.

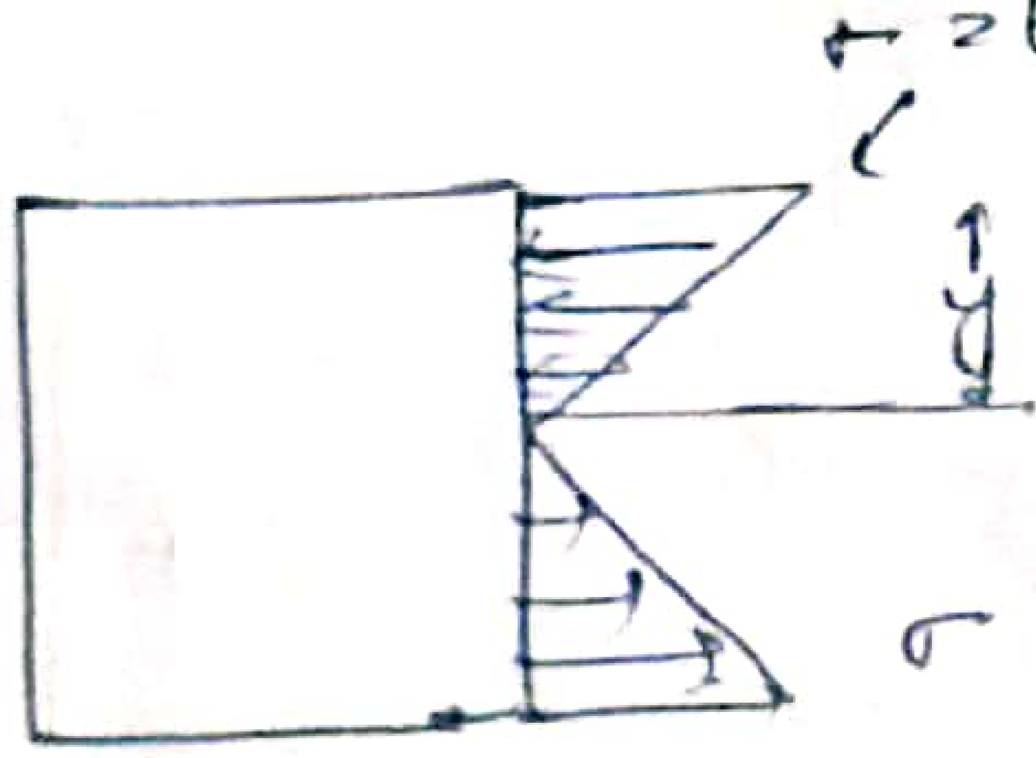
For symmetrical section neutral axis is the axis of symmetry and it always passes through the centre of area (Centroid).

Q> What is the assumptions used in bending?

1> Material is homogenous, isotropic, elastic and hooks law is valid.

2> Young's Modulus of Elasticity is same for tension & compression.

$\sigma = E_t \epsilon$



$\epsilon \propto y$

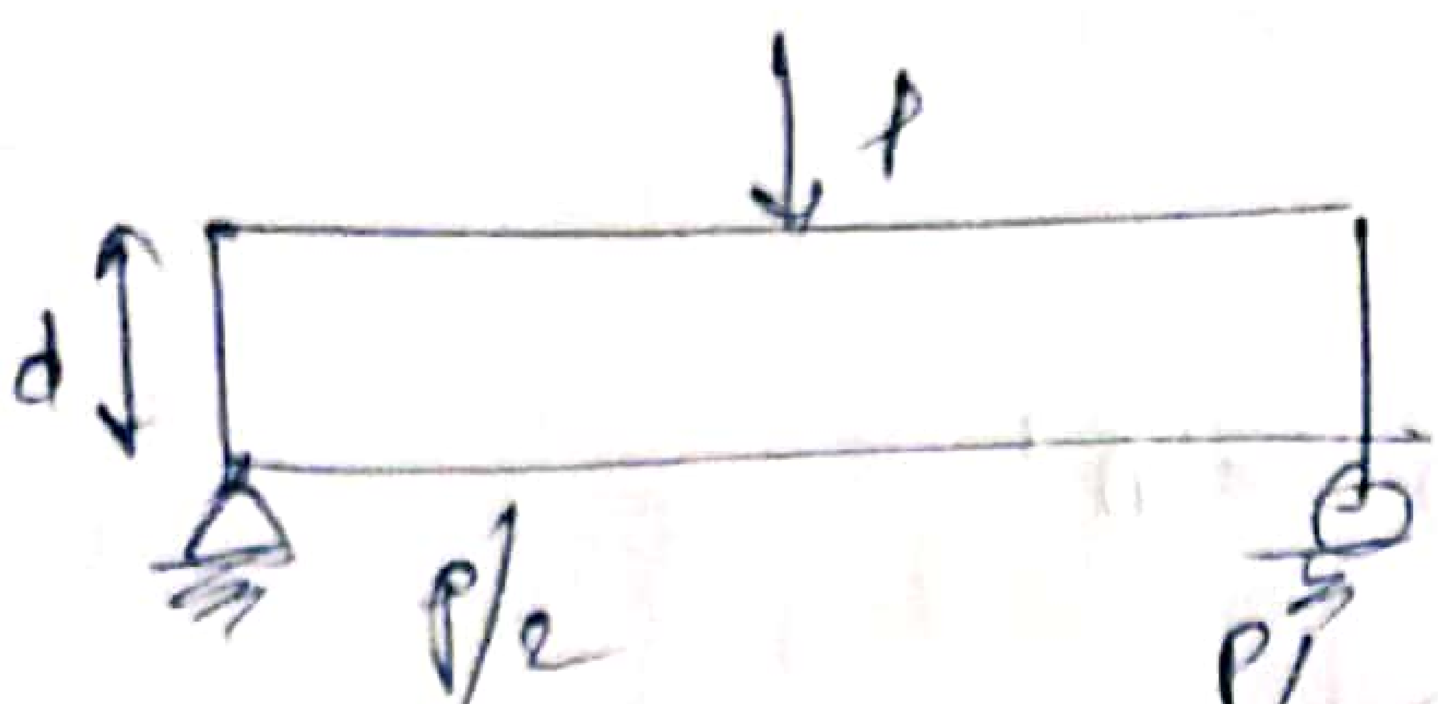
$\sigma = E\epsilon$

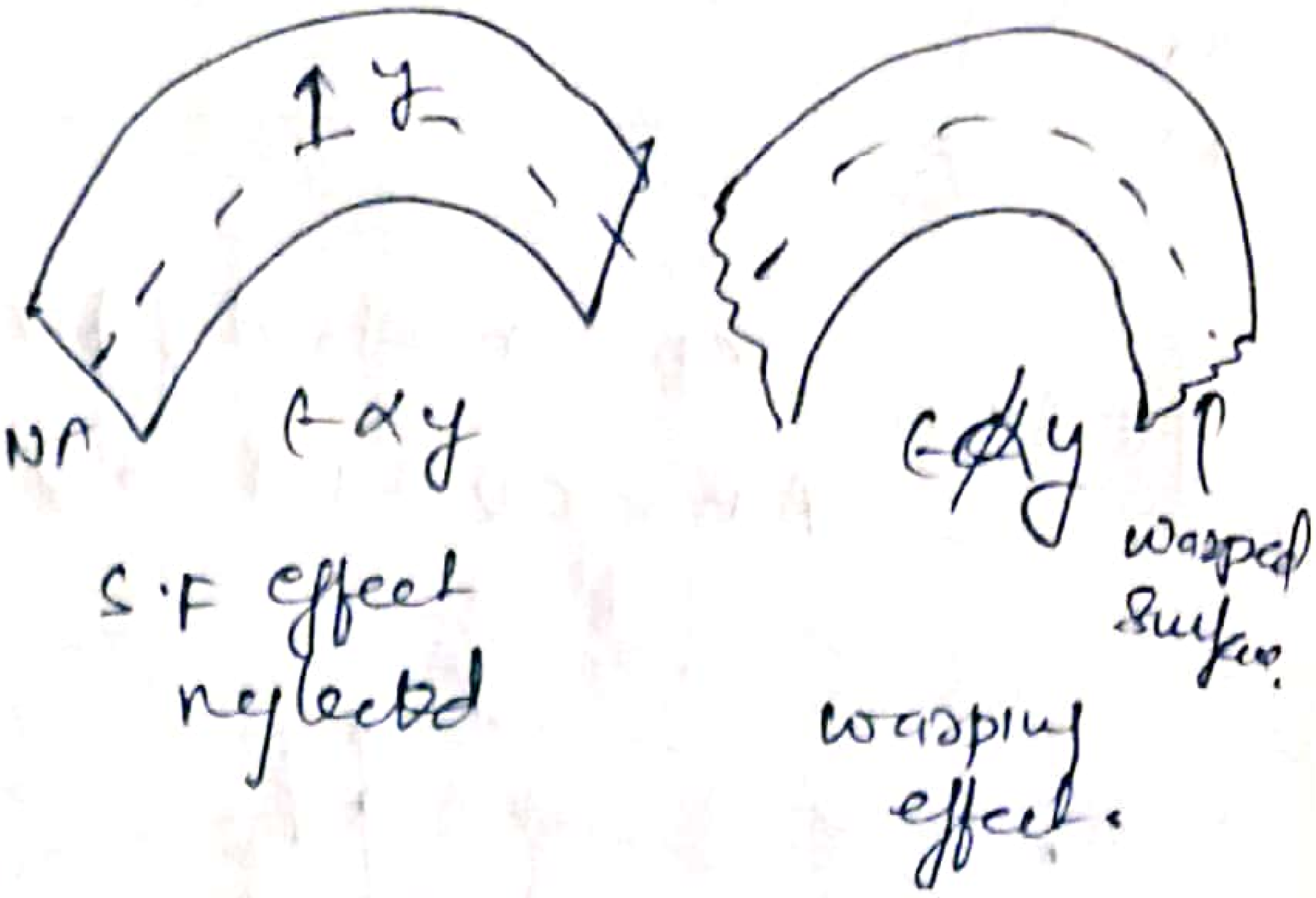
$\sigma = E_c \epsilon$

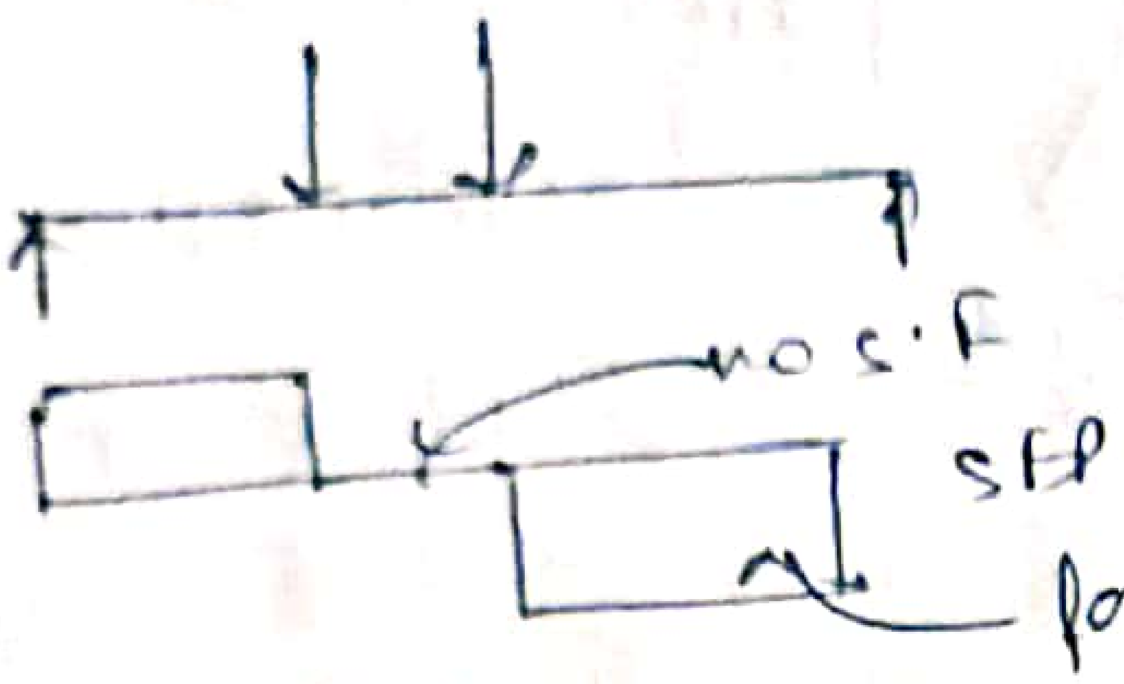
$\frac{\sigma}{\epsilon} = E$

3> plane cross-section remains plane before and after bending. (Plane Surface is obtained when only pure B.M is applied or S.F effect is neglected). In that case strain is

proportional to y ($\epsilon \propto y$).
If designer used shear force (generally for high depth beam) then warped surface will be obtained and ($\epsilon \not\propto y$). (Not proportional)

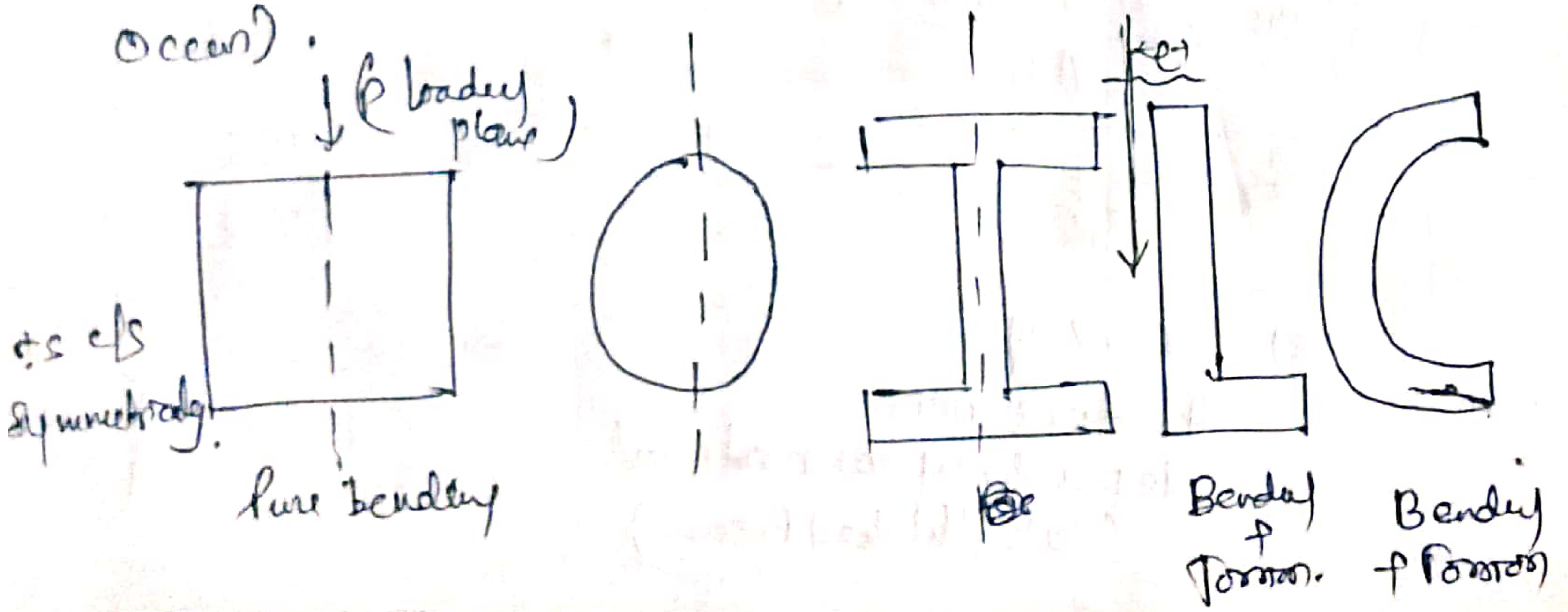


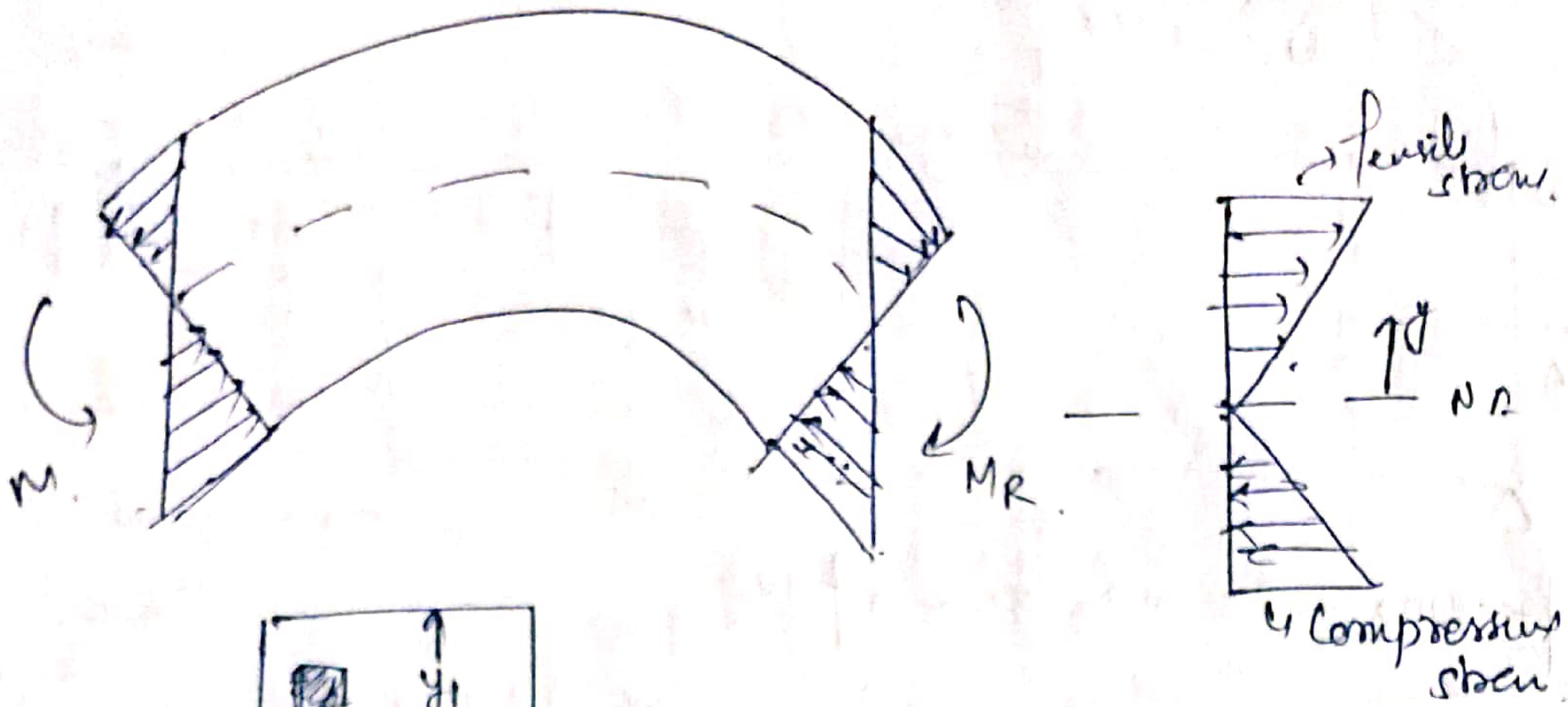


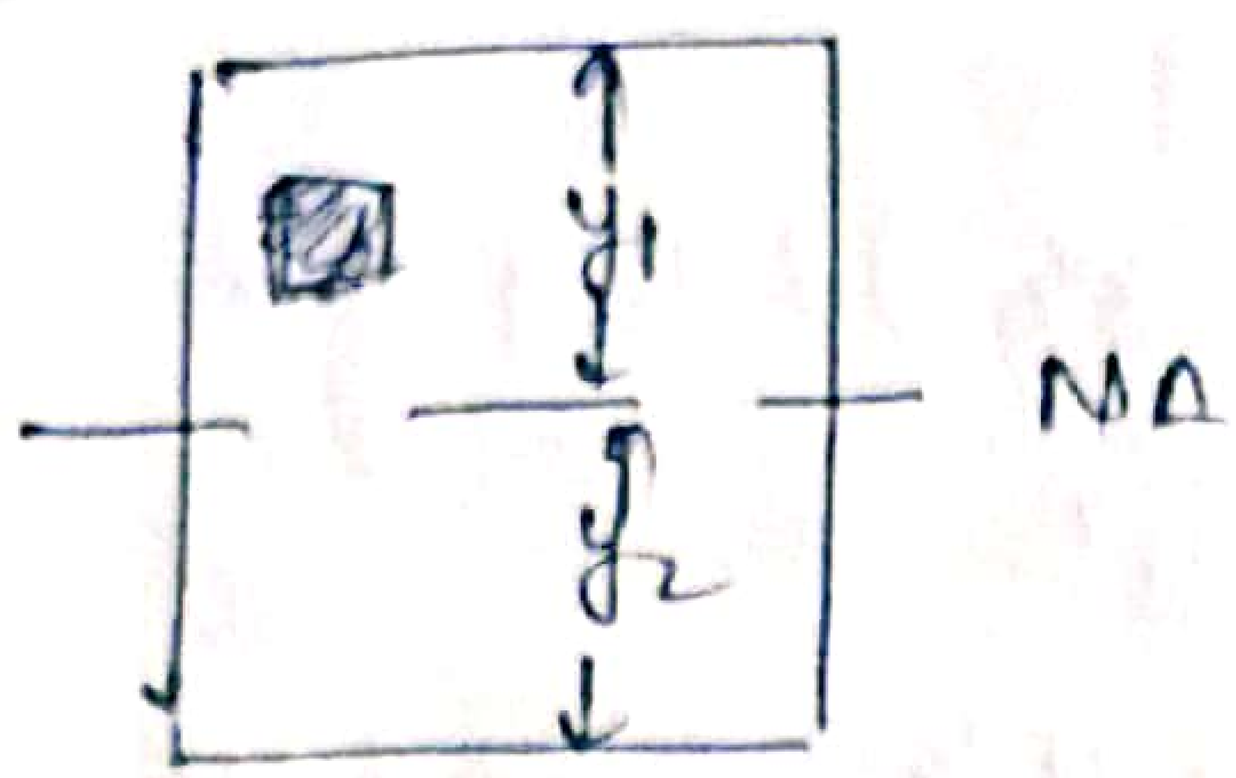


for this section designing assumption is used.

4) Section is prismatic and symmetric in loading plane. If section is not symmetric then bending and torsion both will occur. At that time designer calculate shear centre (shear centre is a point, if load is passing from that point no twisting will occur).







$y_1 = y_2$ = Symmetric Section.

$y_1 \neq y_2$ = Non-symmetric section

$I_{NA} \rightarrow$ M.O.I about neutral axis

And by proof we get Euler Bernoulli Bending Eqn.

$$\boxed{\frac{M_R}{I_{NA}} = \frac{\sigma}{y} = \frac{E}{R}}$$

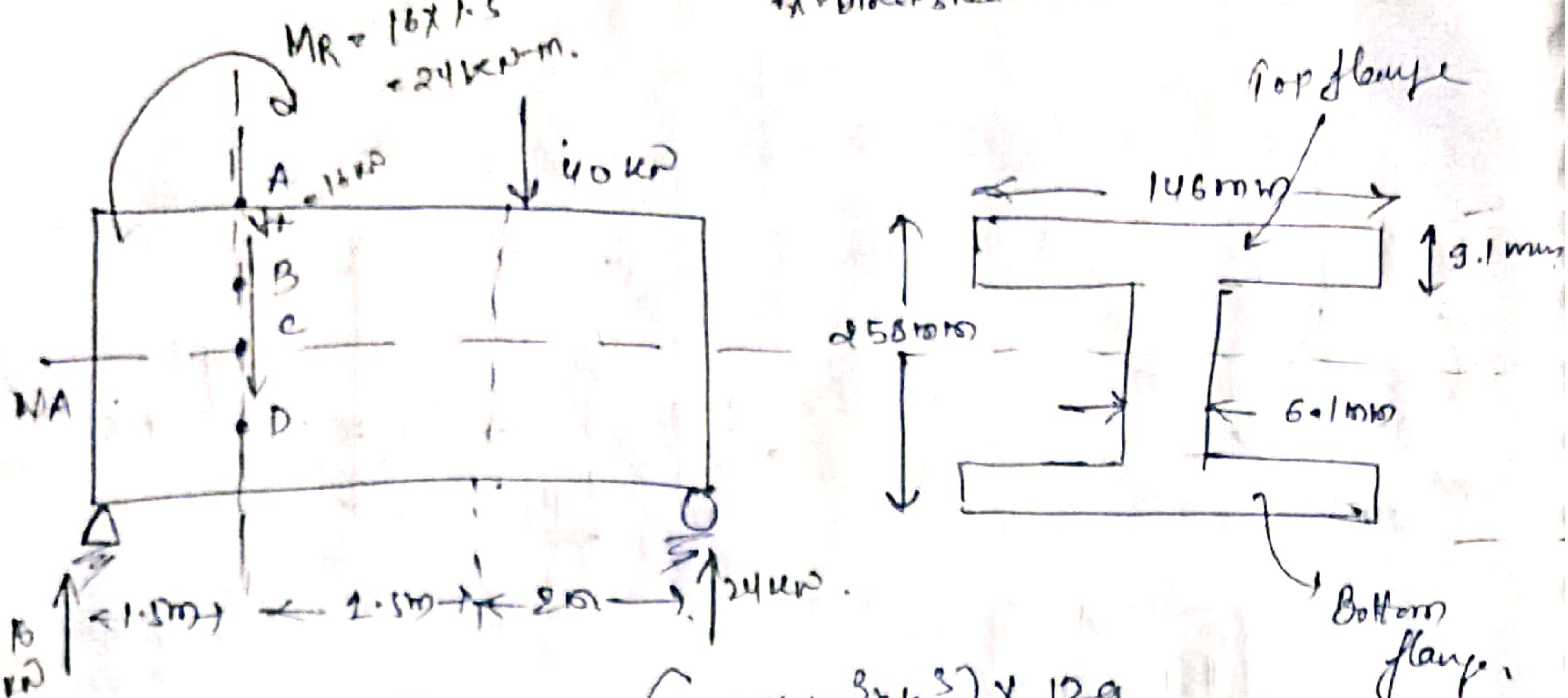
& $M_R = M$

Q. A S.S.B having I c/s of length 5m in which transverse load 40 kN is acting as shown in fig find out the flexural stresses at pt. A, B, C, D.

$I_{NA} = 49.1 \times 10^6 \ mm^4$

Bending is a normal stress

τ_A = Direct shear stress is a shear stress.



So $\sigma_A = \frac{M}{I} y_A = \frac{(24 \times 10^3 \times 10^3) \times 129}{49.1 \times 10^6}$

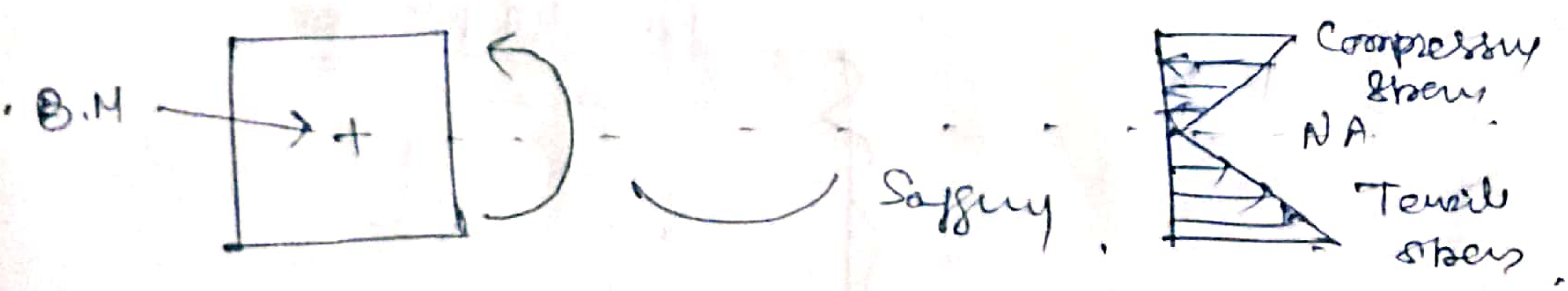
$= 63.055$ MPa

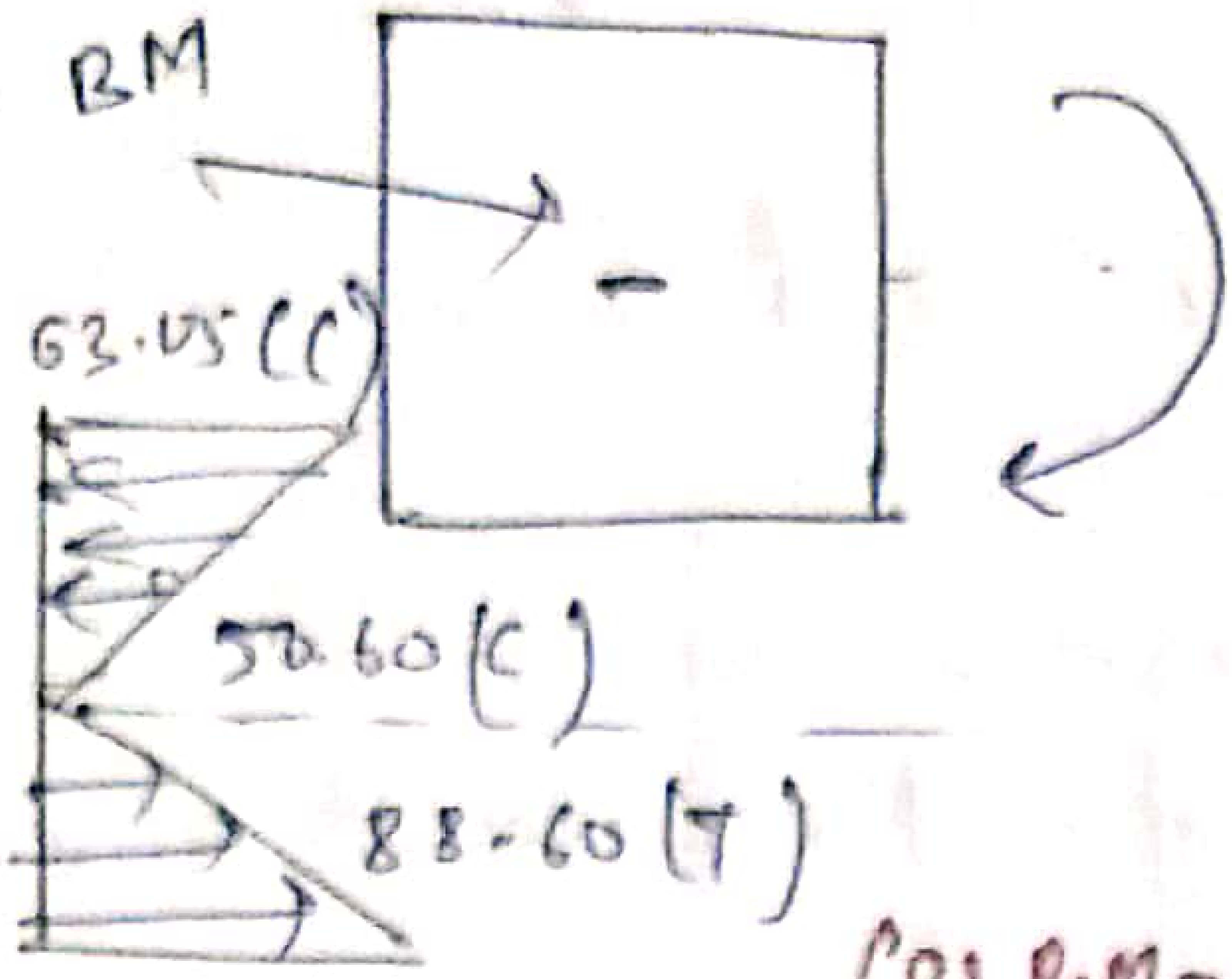
$\sigma_B = \frac{M y_B}{I} = \frac{(24 \times 10^3 \times 10^3) \times 119.9}{49.1 \times 10^6} = 58.607$ MPa

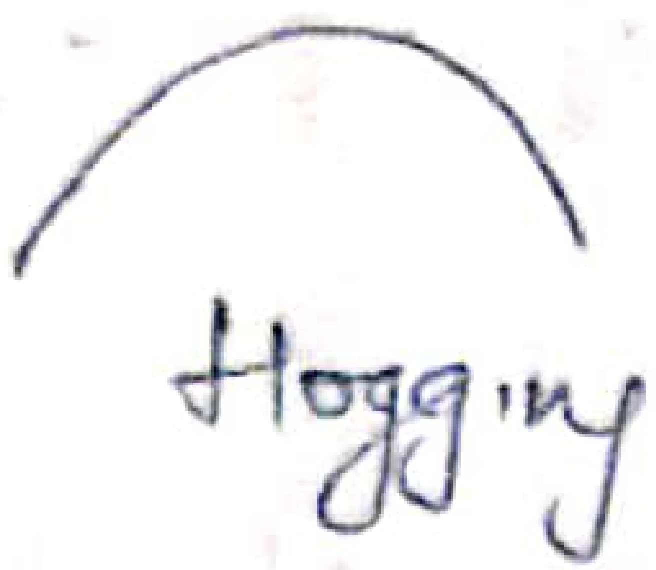
$\sigma_C = \frac{M}{I} y_c = 0$

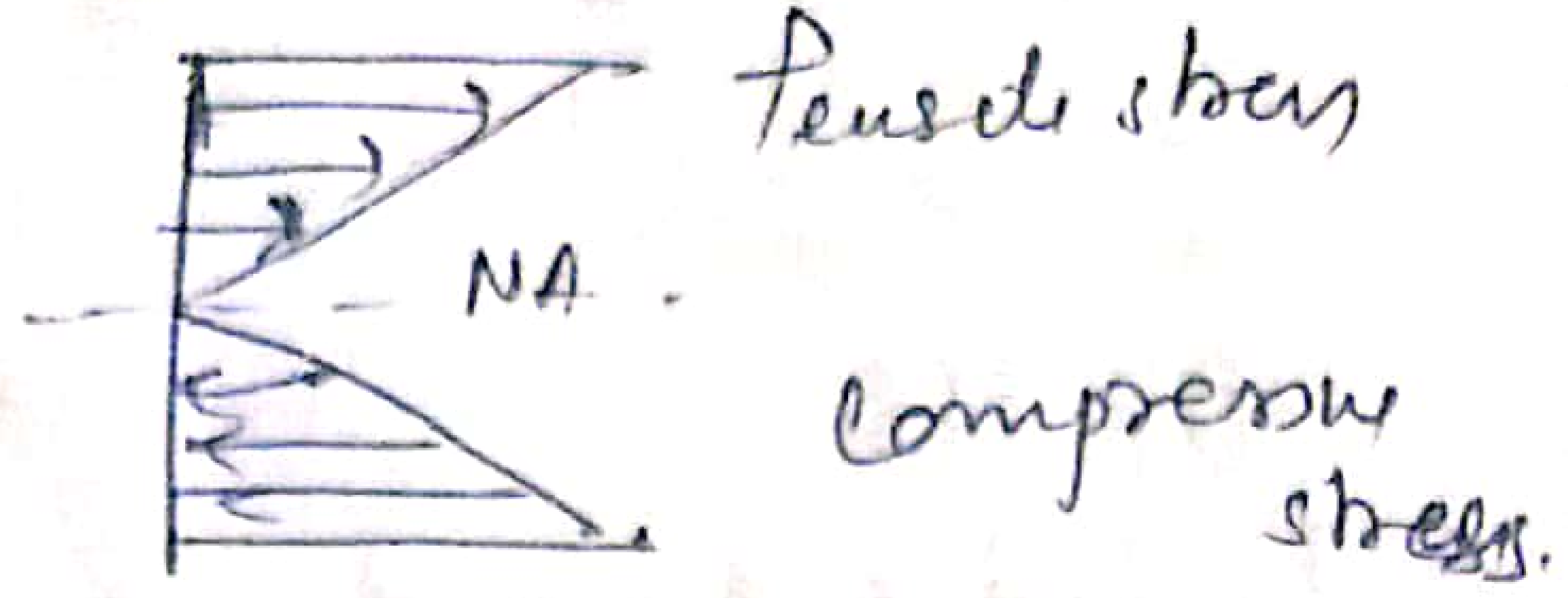
$\sigma_D = \frac{M y_D}{I} = \frac{(24 \times 10^3 \times 10^3) \times 119.9}{49.1 \times 10^6} = 58.607$ MPa

Sign Convention: Left to right.



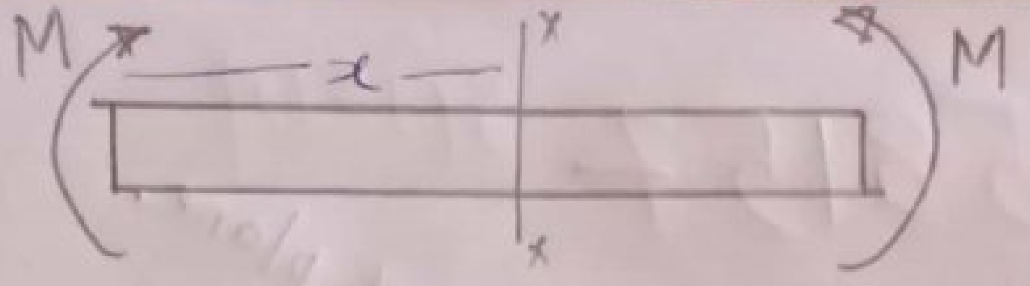






↳ tensile stress.

(as B.M = +24 kN-m)



$SF_x = 0$

* <u>Neutral Fiber:-</u> It is the fiber at which net force is zero, hence elongation of fibers is zero, so stress and strain are also zero

* Neutral Axis (NA):- It is defined as the line intersection between plane of cross section and neutral fibers.

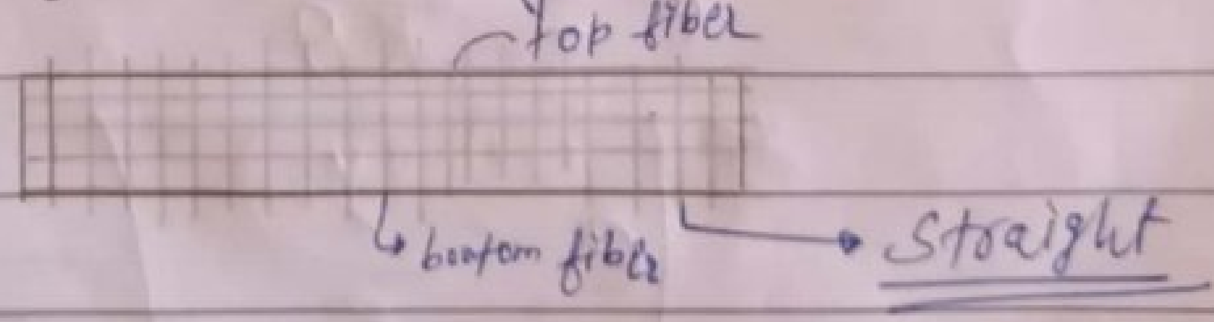
✡ <u>Theory of Bending:-</u> (pure bending, SF = 0)

<u>Assumption:-</u>

→ Beam is initially straight
→ Homogeneoution & isotropic material of Beam.
*→ The material of beam obey's Hookes low..
  ↳ linear elastic solid
  ↳ (T = Const., No. stress Consantration)

**→ Self weight of component is not consider.

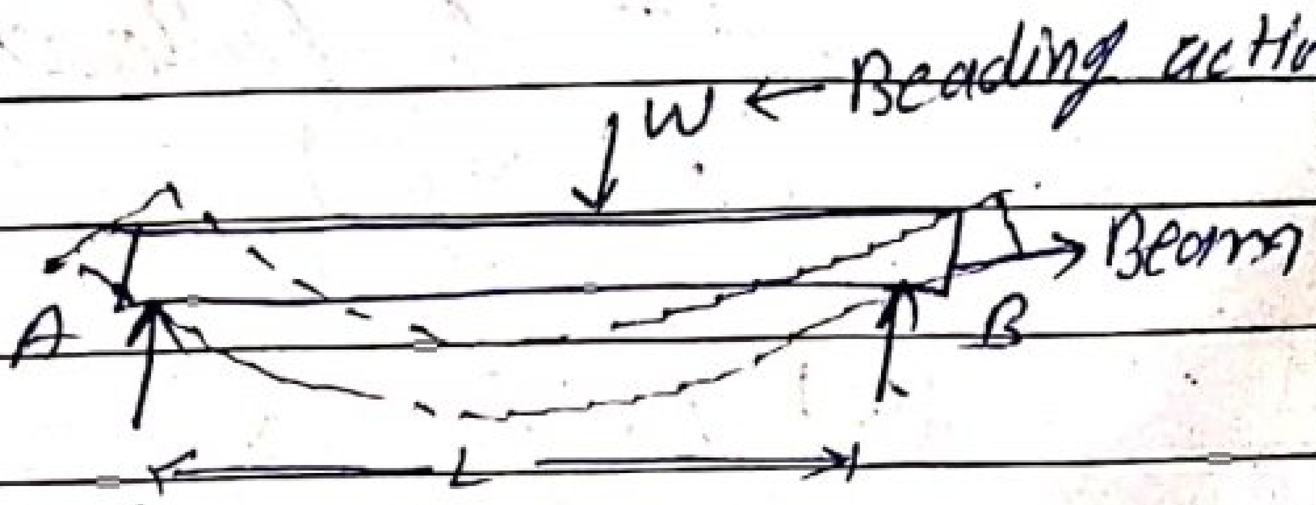
*→ plane transverse ~~section~~ fiber will remain plane even after bending.



*→ poisson's effect is neglected.
→ young's modulus of elasticity is same for tension and compression.
→ Beam is prismetic (A_c = Const)
→ Beam cross section have axis of

Stresses in beams

Bending stresses:-

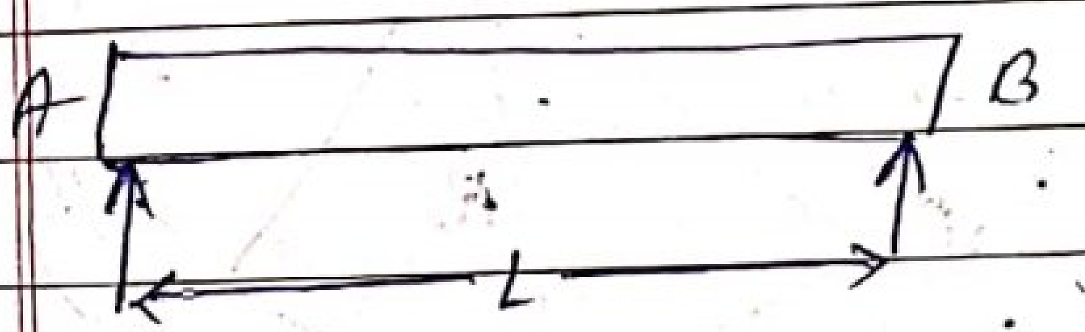


→ Bending stresses are the internal resistance to external force which causes bending of a member

→ it is denoted by 'σ_b'

→ it's unit will be 'N/mm^2'

→ Bending of simply supported & cantilever beam.



(i) simply supported Beam

(ii) cantilever beam.

Assumptions made in the theory of simple bending:-

(a) material of the beam is homogenous & isotropic.

(b) Beam is straight before loading & remains straight even after load is removed.

(c) Beam is stressed within elastic

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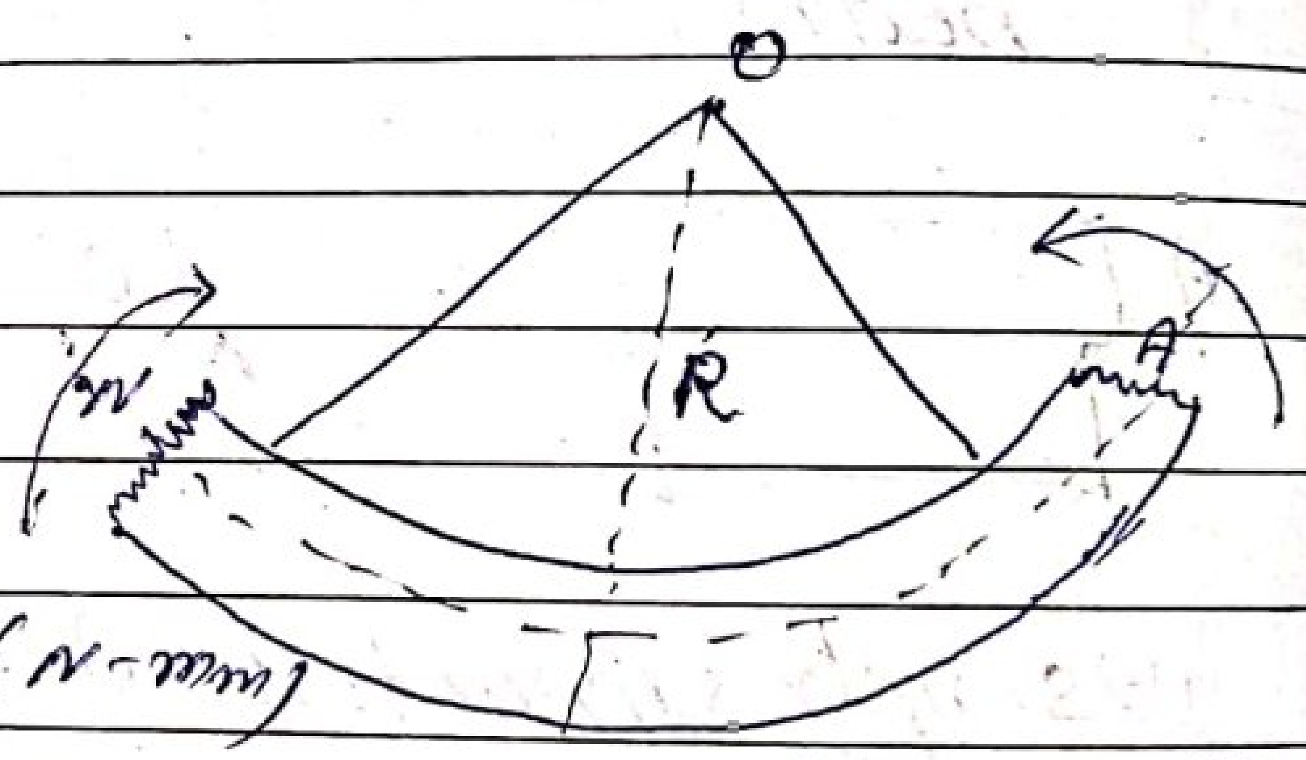
limit & follows Hooke's law.
($\sigma \propto e$)

(iv) Beam is subjected to pure bending. i.e, shear stress are not to be consider.

(v) The layer's at neutral axis does not take bending action.

Flexural Bending formula :-

$$\boxed{\frac{M}{I} = \frac{\sigma_b}{y} = \frac{E}{R}}$$



where:-

M = Bending moment (N-mm)

I = M.O.I for beam - cross . section (mm^4)

σ_b = Bending stress (N/mm^2)

y = distance b/w neutral axis to extreme fibre.

E = Young's modulus (modulus of elasticity for beam material (N/mm^2)

R = Radius of curvature of beam

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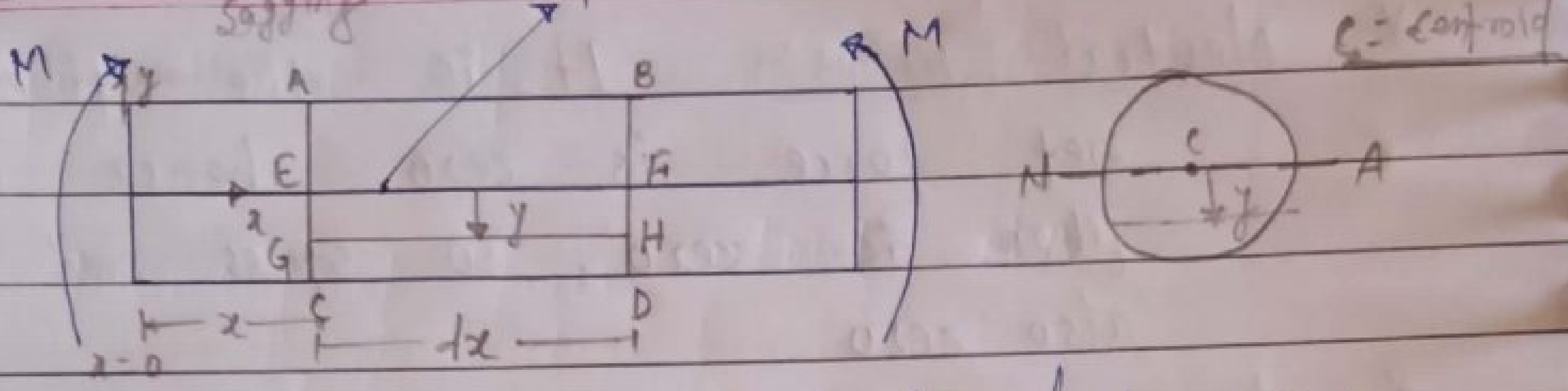
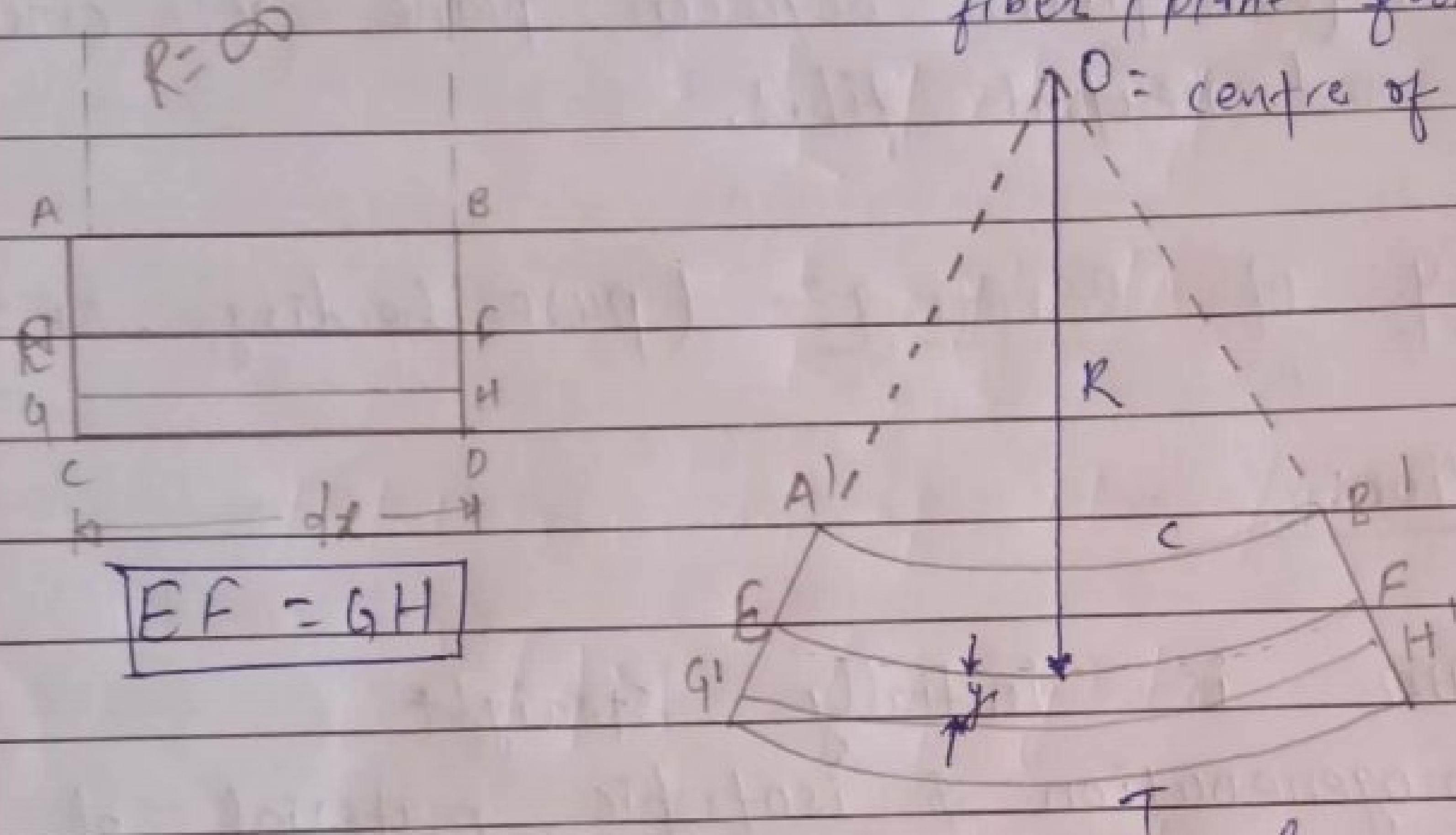
Theory:-



y = perpendicular distance any fiber / plane from N.A



$\boxed{EF = GH}$

"strain fiber is longitudinal strain"

strain in a plane at a distance 'y' from neutral axis

Arc = $\frac{\text{angle}}{R}$

<u>Strain variation:-</u>

$$\text{strain}(e) = \frac{G'H' - GH}{GH}$$

$EF = R\theta = GH$

$G'H' = (R+y)\theta$

$$e = \frac{(R+y)\theta - R\theta}{R\theta}$$

$$e = \frac{y\theta}{R\theta}$$

$$* \boxed{e = \frac{y}{R}} \quad \text{—— (I)}$$

<u>Stress variation:</u>

$$\frac{\text{stress}(\sigma)}{\text{strain}(e)} = E$$

$$\boxed{e = \frac{\sigma}{E}} \quad \text{—— (II)}$$

∴ <u>E = young modulus.</u>

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From eqⁿ (I) & (II)

$$\boxed{\frac{\sigma}{E} = \frac{y}{R}}$$

$$\boxed{\frac{\sigma}{y} = \frac{E}{R}} \quad \text{(III)}$$

σ = Bending stress. $= \sigma_b$

$$\sigma_b = \left(\frac{E}{R}\right) \times y.$$

$\therefore \frac{E}{R} = \text{Constant}$

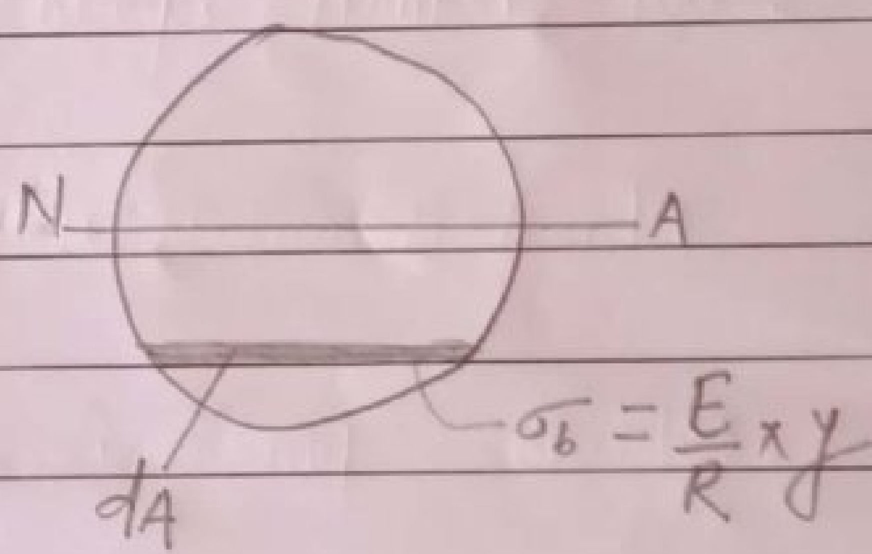
$$\sigma_b \propto y$$

The variation of bending stress is linear

$y = 0 \quad \sigma_b = 0$

Resisting force $= \sigma_b \times dA$

$= \frac{E}{R} y \, dA$



Total force on the beam section is obtaind by integrating above eqⁿ.

$$= \int \frac{E}{R} y \, dA$$

$$= \frac{E}{R} \int y \, dA$$

In pure bending $F = 0$

$$\frac{E}{R} \int y \, dA = 0$$

$$\boxed{\int y \, dA = 0}$$

→ moment of area dA about

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Resisting moment:-

Moment of force about NA

$$dM = \text{Force on layer} \times y$$

$$= \frac{E}{R} y \times dA \times y$$

$$= \frac{E}{R} \times y^2 \times dA$$

Total moment of the force on the section

$$\int dM = \int \frac{E}{R} y^2 dA$$

$$M = \frac{E}{R} \int y^2 dA$$

$\therefore \int y^2 dA$ repersent the moment of inertia.

$$M = \frac{E}{R} \times I$$

$$\boxed{\frac{M}{I} = \frac{E}{R}} \quad \text{—— (iv)}$$

From eqn (iii) & (iv)

$$\boxed{\frac{M}{I} = \frac{E}{R} = \frac{\sigma}{y}}$$

→ Bending equation

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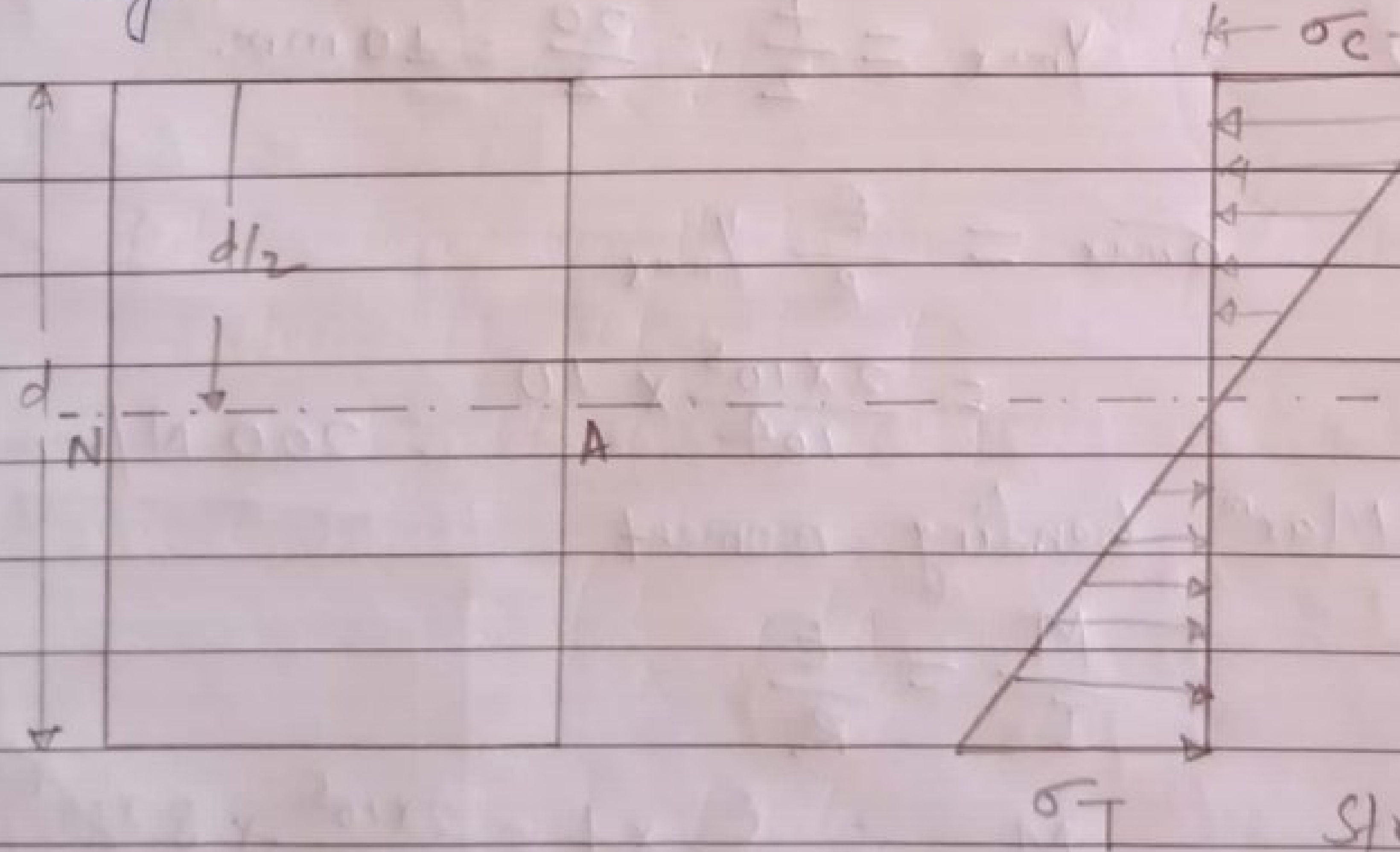
To find Max^m Bending moment

$$\boxed{(\sigma)_{max} = \frac{M_{max}\, y}{I}}$$

☆ Bending stresses In Symetrical Section:-

The neutral axis of a Symetrical Section (circular, Rectengular or square) lies at a distance of $d/2$ from the outermost layer of section.

↳ There is no stress at the neutral axis.
↳ But the stress at a point is directly proportional to its distance from the neutral axis.
↳ The max^m stress take place at the outermost layer.



Stress diagram

Q. A steel plate of width 120 mm and of thickness 20mm is bent into a circular arc of Radius 10 m. Determine the max^m stress induced and bending moment will produce the max^m stess. $E = 2\times10^6\ N/mm^2$

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Sol^n:- width (d) = 120 mm.
$t = 20$ mm

moment of inertia $I = \frac{bt^3}{12}$
$= \frac{120 \times (20)^3}{12}$
$= 10 \times 8000 = 80000$
$= 8 \times 10^4$ mm^4

$R = 10$ m $= 10000$ mm
$= 10^{\cancel{4}}$ mm.

Young modulus (E) = 2×10^5 N/mm^2

Now,
$$\frac{E}{R} = \frac{\sigma}{y}$$
$$\sigma = \frac{E}{R} \times y$$
$\therefore y_{max} = \frac{t}{2} = \frac{20}{2} = 10$ mm.

$$\sigma_{max} = \frac{E}{R} y_{max}$$
$= \frac{2 \times 10^5}{10^4} \times 10$ $= 200$ N/mm^2 Ans

(b) Max^m bending moment
$$\frac{M}{I} = \frac{E}{R}$$
$M_{max} = \frac{E}{R} \times I = \frac{2 \times 10^5}{\cancel{10^4}} \times 8 \times \cancel{10^4}$
$= 16 \times 10^5$ N-mm

Ans

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✡ SECTION MODULUS :-

Section modulus is defined as the ratio of moment of inertia of the section about neutral axis to the distance of the outermost layer from the neutral axis.

It is denoted by 'Z'.

$$\boxed{Z = \frac{I_{NA}}{y_{max}}}$$

Now,

$$\frac{M}{I} = \frac{\sigma}{y}$$

The stress(σ) will be max^m, when y is max^m.

$$\frac{M}{I} = \frac{\sigma_{max^m}}{y_{max^m}}$$

$$M = \sigma_{max^m} \frac{I}{y_{max^m}}.$$

$$\boxed{M = \sigma_{max^m} \times Z}$$

→ M is the max^m. bending moment. Hence, moment of resistance offered the section is max^m. when section modulus Z is max^m. Hence section modulus repessent the <u>strength</u> of the section.

→ Greater the value of Z, greter the bending strength.

→ The value of Z depends upon moment of inertia and distribution of area.

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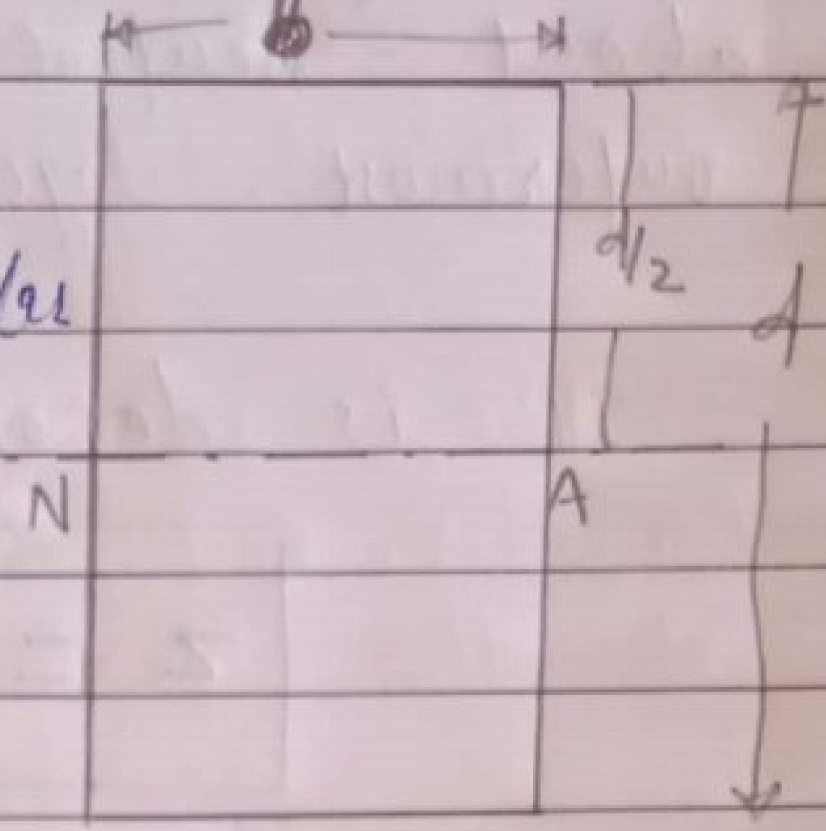
Section Modulus for Various shapes OR Beam section:-

1. Rectengular Section.



Moment of inertia of a rectengular section about an axis through its C.G (N.A)

$$I = \frac{bd^3}{12}$$

Distance of outermost layer

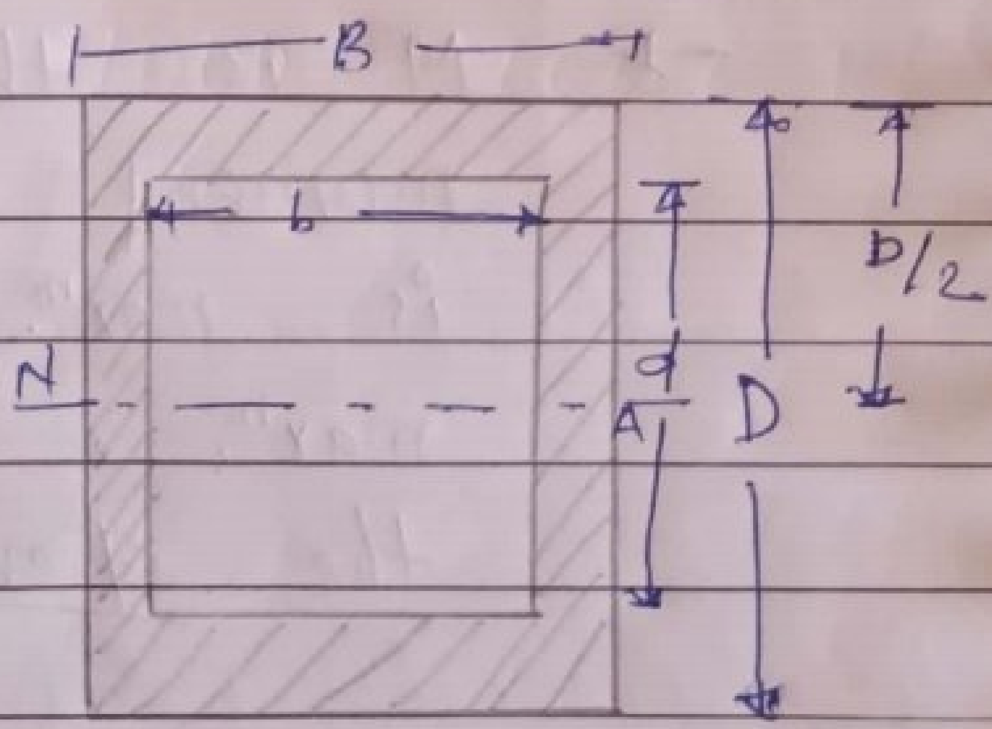
$$y_{max} = d/2$$

$$\text{Section modulus } (Z) = \frac{I}{y_{max}}$$

$$= \frac{\frac{bd^3}{12}}{\frac{d}{2}} = \frac{bd^3}{12} \times \frac{2}{d}$$

$$= \frac{bd^2}{6}$$

2. Hollow Rectangular Section:-



$$I = \frac{BD^3}{12} - \frac{bd^3}{12}$$

$$= \frac{1}{12}(BD^3 - bd^3)$$

$$y_{max} = D/2$$

$$Z = \frac{I}{y_{max}} = \frac{\frac{1}{12}(BD^3 - bd^3)}{D/2} = \frac{1}{6D}(BD^3 - bd^3)$$

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3. <u>Circular section</u>:-

$$I = \frac{\pi}{64} d^4$$

$$y_{max} = \frac{d}{2}$$

$$Z = \frac{I}{y_{max}} = \frac{\pi d^4}{64} \times \frac{2}{d} = \frac{\pi d^3}{32}$$

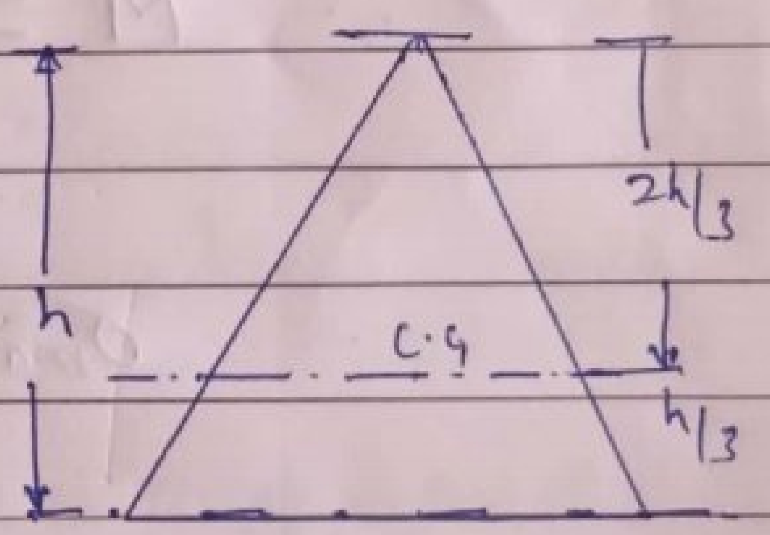
⇒ In Case of polar moment of inertia.

$$I_p = I_{xx} + I_{yy}$$

$$= \frac{\pi d^4}{64} + \frac{\pi d^4}{64} = \frac{\pi d^4}{32}$$

$$Z_p = \frac{\frac{\pi d^4}{32}}{d/2} = \frac{\pi d^3}{16}$$

4. <u>Triangular Section</u>:-



$$I_{C.G} = \frac{1}{36} bh^3$$

$$I_{base} = \frac{1}{12} bh^3$$

$$y_{max} = \frac{2h}{3}$$

$$Z = \frac{I_{C.G}}{y_{max}} = \frac{\frac{1}{36} bh^3}{\frac{2h}{3}} = \frac{bh^3}{\cancel{36}_{12}} \times \frac{\cancel{3}}{2\cancel{h}}$$

$$= \frac{bh^2}{24}$$

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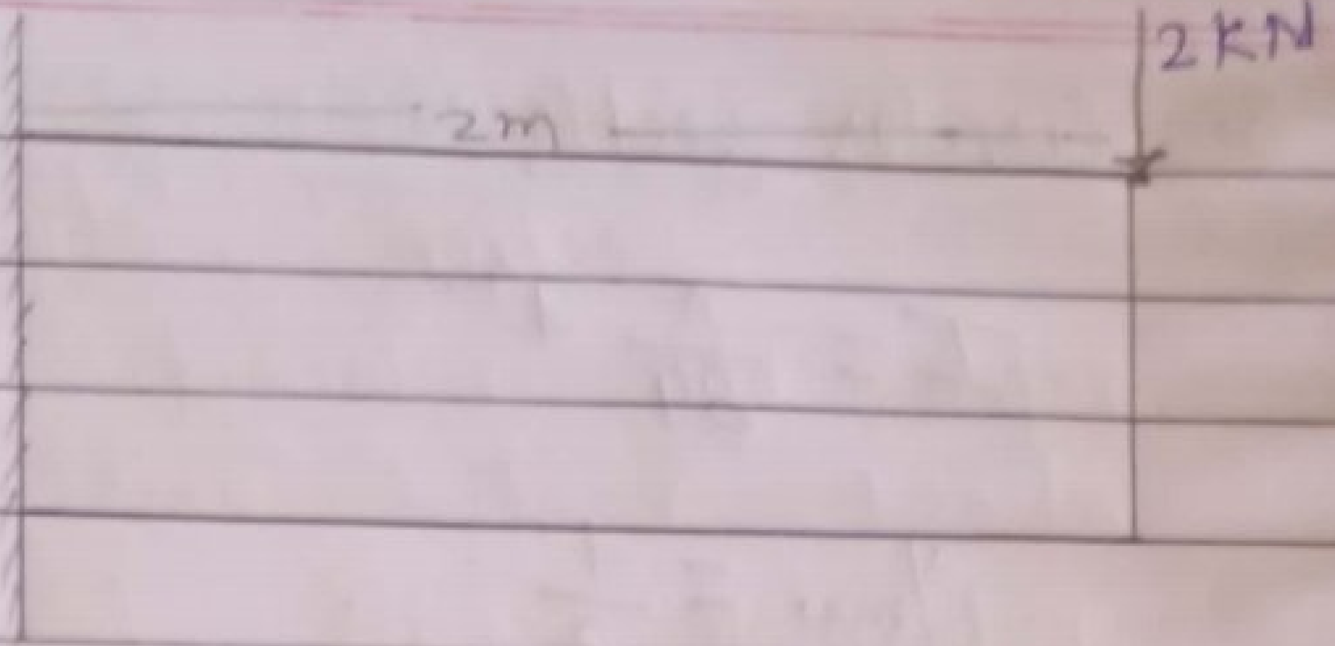
Q.3 Solⁿ:- $L = 2m.$

$W = 2kN$

beam $= \underset{b}{40} \times \underset{d}{60}\ mm$

Rectangular beam.



$$I = \frac{bd^3}{12}$$

$$= \frac{40 \times (60)^3}{12} = 40 \times 5 \times 3600 = 720000$$

$$y_{maxm} = \frac{d}{2} = \frac{60}{2} = 30\ mm$$

$$\text{Section modulus}(Z) = \frac{I}{y_{max}} = \frac{720000}{30}$$

$$= \underline{24000\ mm^3}$$

$$M = \sigma_{max} \times Z$$

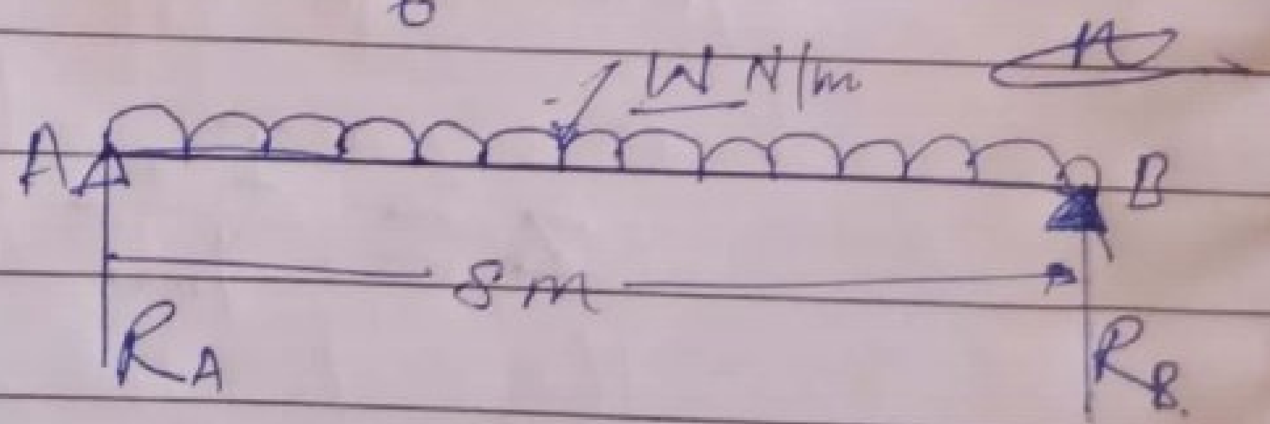
$$\therefore M = W \times L = 2000 \times 2000$$

$$= 4 \times 10^6\ N\text{-}mm$$

$$\sigma_{max} = \frac{M}{Z} = \frac{\cancel{4} \times 10^6}{\underset{6}{\cancel{24}} \times 10^3}$$

$$= \frac{1000}{6} = \underline{166.6\ N/mm^2}$$

Q.4 Solⁿ.



$W = ?$

$d = 200\ mm$

$b = 300\ mm$

$L = 8\ m = 8000\ mm$

$\sigma = 120\ N/mm^2$

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Rectangular section -

$$I = \frac{bd^3}{12} = \frac{300 \times (200)^3}{12} = \frac{300 \times 200 \times 200 \times 200}{12}$$

$$= \frac{2 \times 1\cancel{2} \times 10^8}{\cancel{12}}$$

$$= 2 \times 10^8 \ mm^4$$

$$y_{max^m} = \frac{200}{2} = 100 \ mm.$$

$$Z = \frac{2 \times 10^8}{100} = \underline{2 \times 10^6 \ mm^3}$$

$$M = \sigma_{max} \times Z \qquad \therefore M = \frac{WL^2}{8}$$

$$8000 \times W = 120 \times 2 \times 10^6 \qquad = \frac{W \times 8^2}{8}$$

$$W = \frac{\overset{30}{\cancel{120}} \times \cancel{2} \times 10^6}{\underset{\cancel{4}}{8000}} \qquad = W \times 8$$

$$= \underline{3 \times 10^4 \ N/mm.}$$

✡ BENDING STRESS IN UNSYMMETRICAL SECTIONS:-

→ In Case of Symmetrical Section, the neutral axis passege through the geometrical ~~Section~~ centre of the Section.

→ But in the case of unsymmetrical Section as 'L','T' Sections, the neutral axis does not pass through the geometrical Section of the centre.

→ Hence the value of y for the topmost layer or bottom layer of the section from neutral axis will not be same.

→ For finding the bending stresses in the beam the bigger value of y is used.

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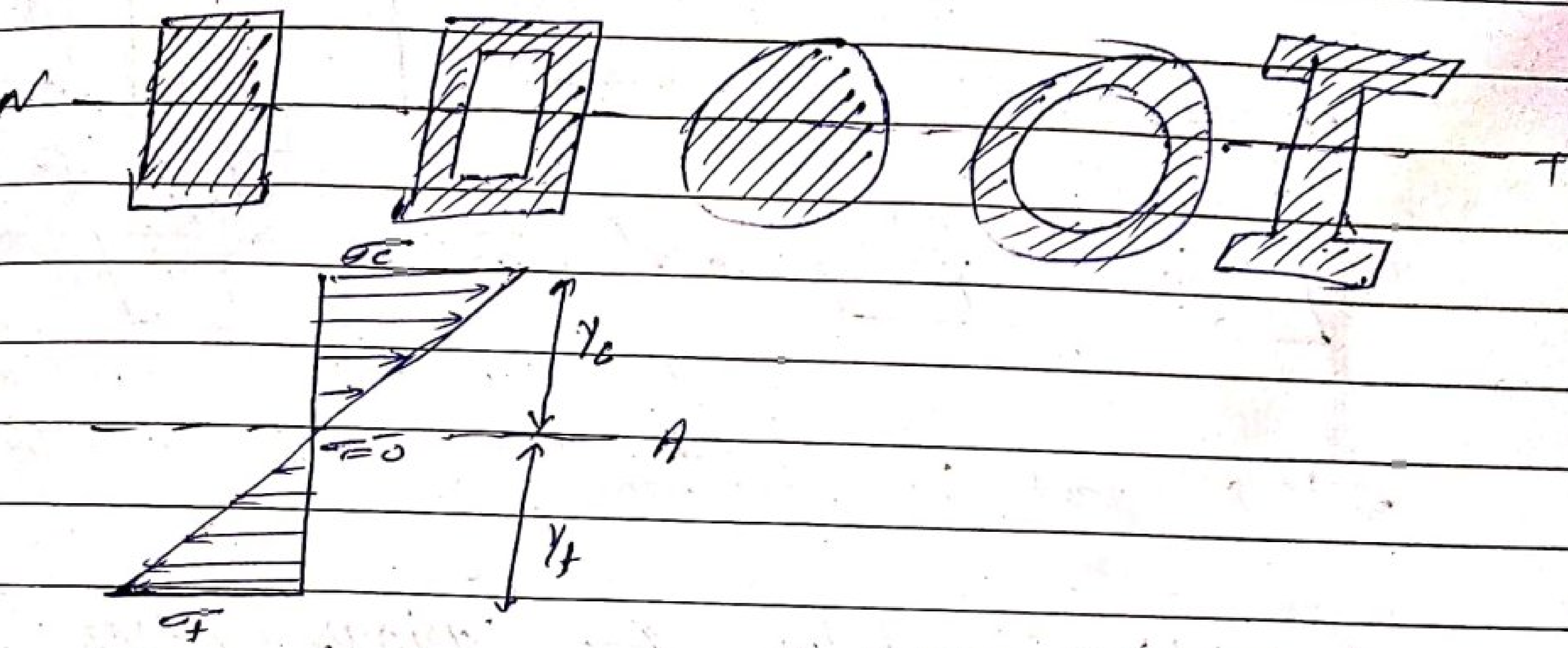
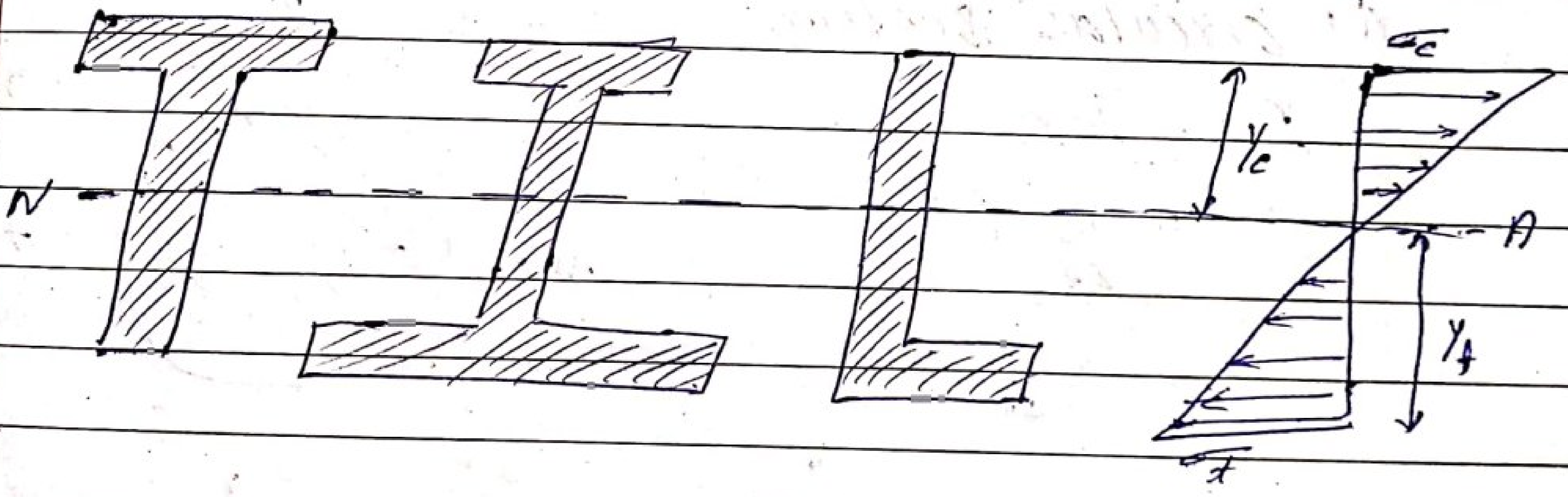
* Bending stress distribution diagrams:-

(i) Symmetric section = $\boxed{Y_c = Y_t}$



(ii) Unsymmetric section = $(Y_c \neq Y_t)$



* Section modulus :-

it is defined as the rate of moment of inertia & distance of layer subjected to max bending from N·A (~~neutral axis~~)

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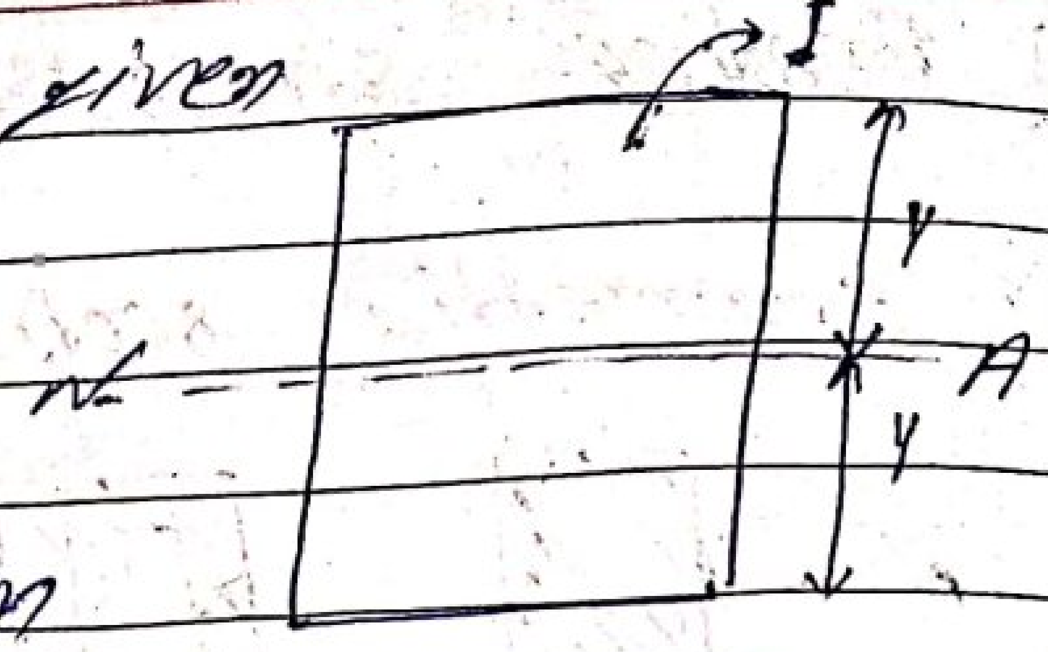
→ section modulus is given by

$$\boxed{Z = \frac{I}{y}}$$



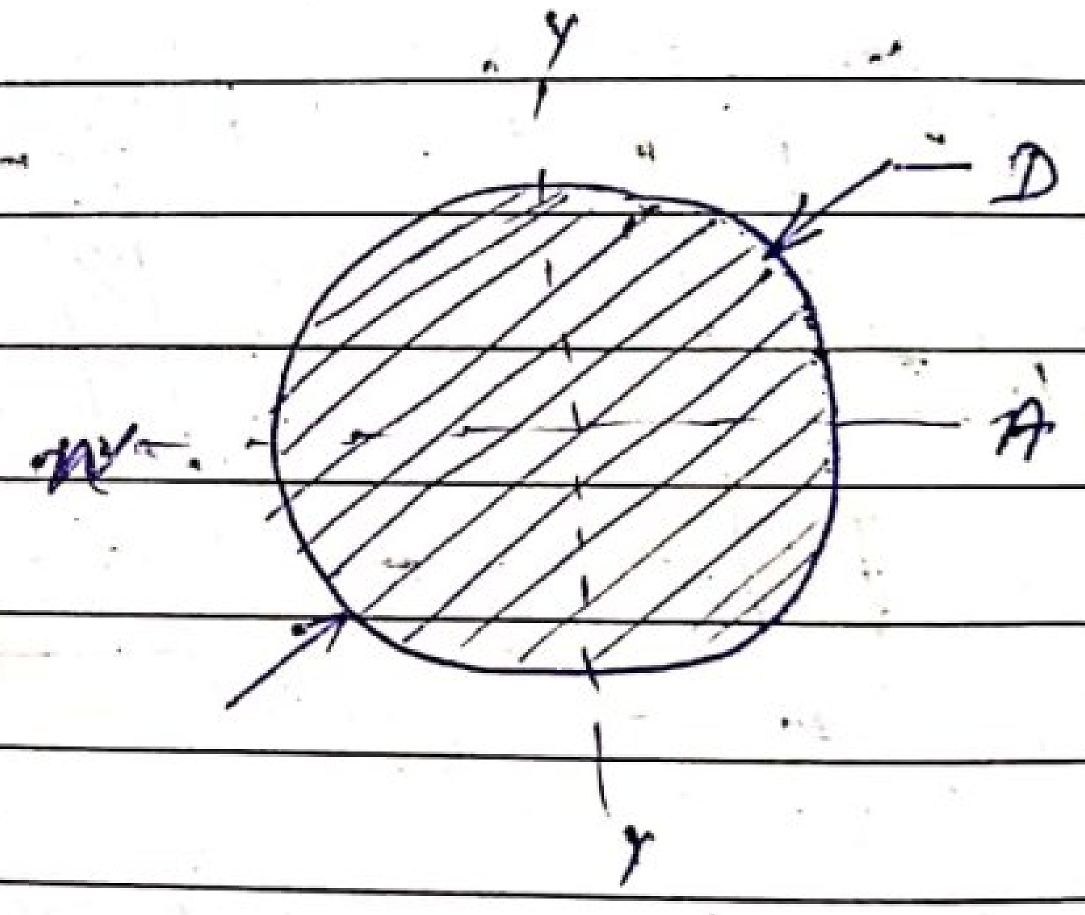
If we know I_{xx}, then

$$\boxed{Z_{xx} = \frac{I_{xx}}{y}} \quad \text{and} \quad \boxed{Z_{yy} = \frac{I_{yy}}{y}}$$

→ it's unit is in 'mm^3'

* section modulus for various cases

(i) circular section:-



$I_{xx} = I_{yy}$

$= \frac{\pi}{64} D^4$

$y_{max} = D/2$

$$\therefore Z_{xx} = Z_{yy} = \frac{I_{xx}}{y_{max}}$$

$$= \frac{\pi}{64} D^4 \div \frac{D}{2}$$

$$= \frac{\pi}{64} D^4 \times \frac{2}{D}$$

$$\boxed{Z_{xx} = Z_{yy} = \frac{\pi D^3}{32}}$$

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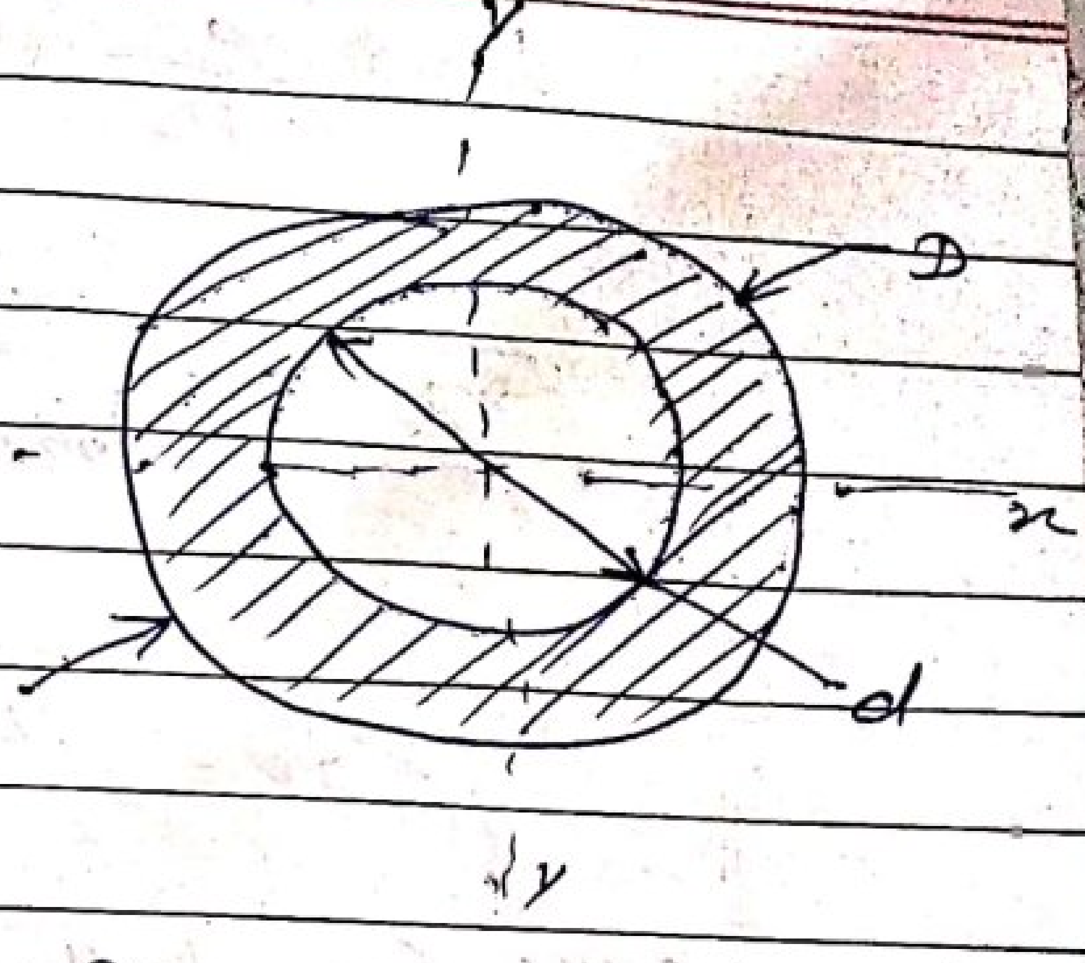
(ii) hollow circular section :-



$$I_{xx} = I_{yy} = \frac{\pi}{64}(D^4 - d^4)$$

$$X_{max} = Y_{max} = \frac{D}{2}$$

$$\therefore Z_{xx} = Z_{yy} = \frac{I_{xx}}{Y}$$

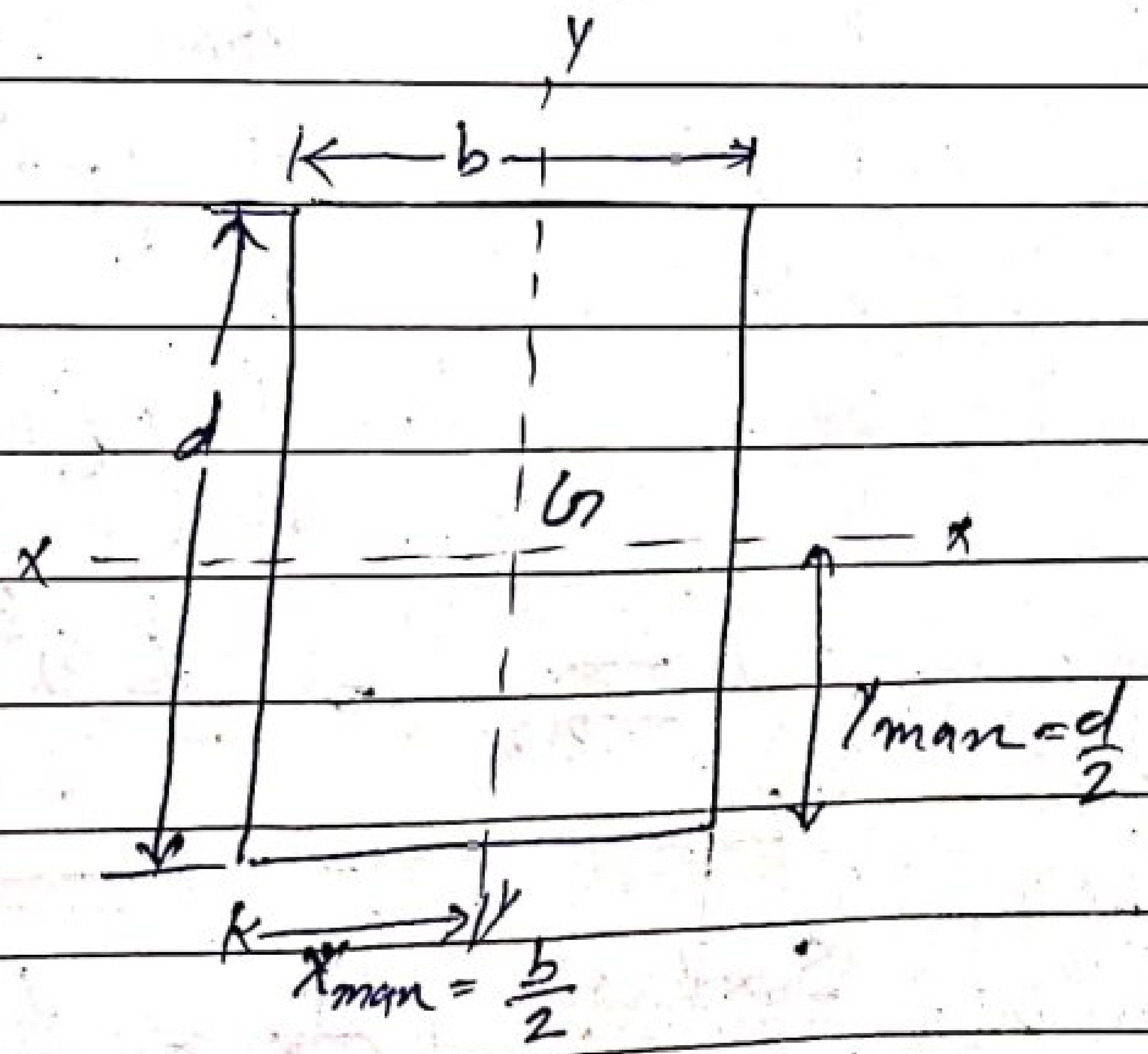
$$= \frac{\pi}{64}(D^4 - d^4) \div \frac{D}{2}$$

$$= \frac{\pi}{64}(D^4 - d^4) \times \frac{2}{D}$$

$$= \frac{\pi}{32}\frac{(D^4 - d^4)}{D}$$

$$\therefore \boxed{Z_{xx} = Z_{yy} = \frac{\pi}{32}\left(\frac{D^4 - d^4}{D}\right)} \; mm^3$$

(iii) Rectangular section :-



$$I_{xx} = \frac{bd^3}{12} \text{ and } Y_{max} = \frac{d}{2}$$

$$\therefore Z_{xx} = \frac{I_{xx}}{Y_{max}}$$

$$= \frac{bd^3}{12} \div \frac{d}{2}$$

$$= \frac{bd^2}{6}$$

Similarly

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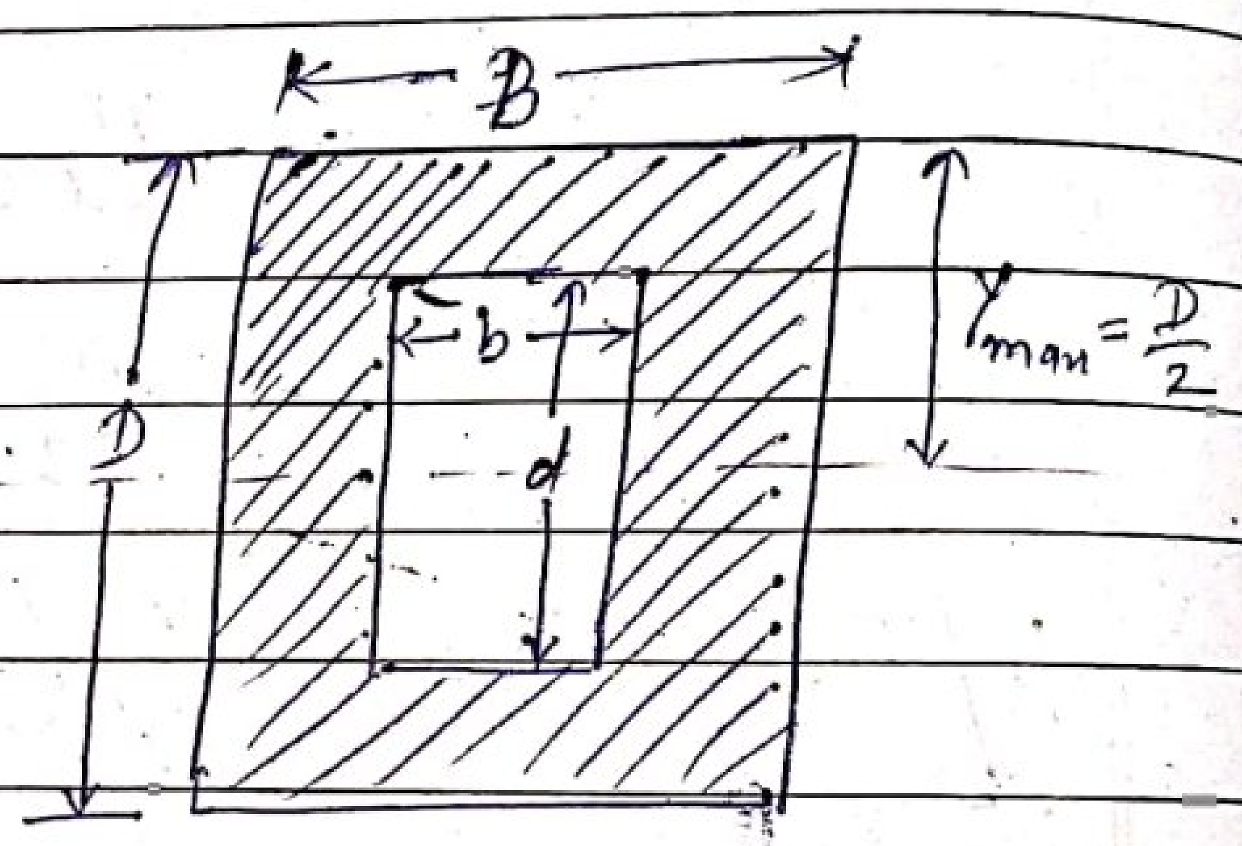
$$I_{yy} = \frac{db^3}{12} \quad \text{and} \quad x_{max} = \frac{b}{2}$$

$$Z_{yy} = \frac{I_{yy}}{x_{max}} = \frac{db^3}{12} \div \frac{b}{2}$$

$$= \frac{db^2}{6}$$

$$\therefore \boxed{Z_{yy} = \frac{db^2}{6}} \ mm^3$$

Q(iii) for hollow rectangular section



$$I_{xx} = \frac{BD^3}{12} - \frac{bd^3}{12}$$

$$= \frac{1}{12}(BD^3 - bd^3)$$

$$Y_{max} = \frac{D}{2}$$

$$\therefore Z_{xx} = \frac{I_{xx}}{Y_{max}}$$

$$= \frac{1}{12}(BD^3 + bd^3) \div \frac{D}{2}$$

$$= \frac{1}{12}(BD^3 - bd^3) \times \frac{2}{D}$$

$$\boxed{Z_{xx} = \frac{(BD^3 - bd^3)}{6D}}$$

Similarly $\boxed{Z_{yy} = \frac{DB^3 - db^3}{6B}}$

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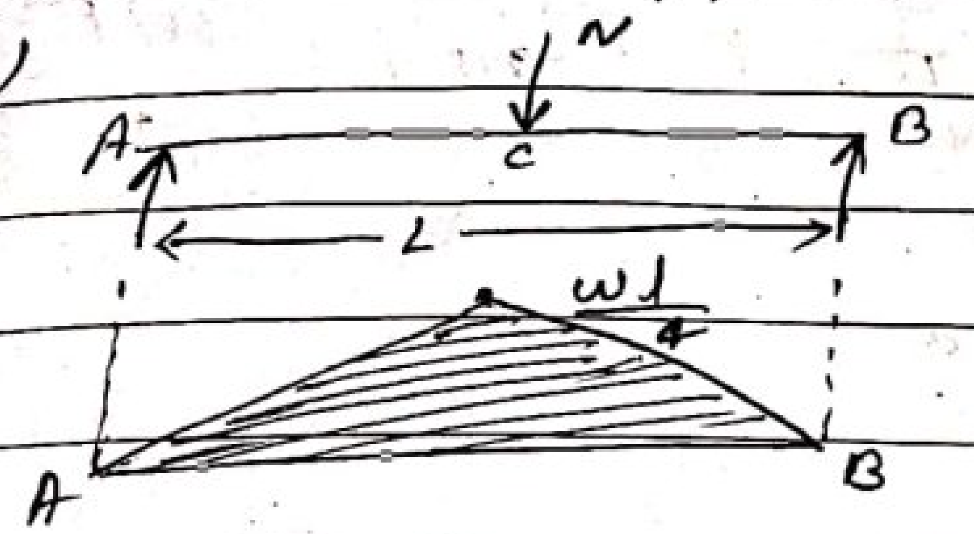
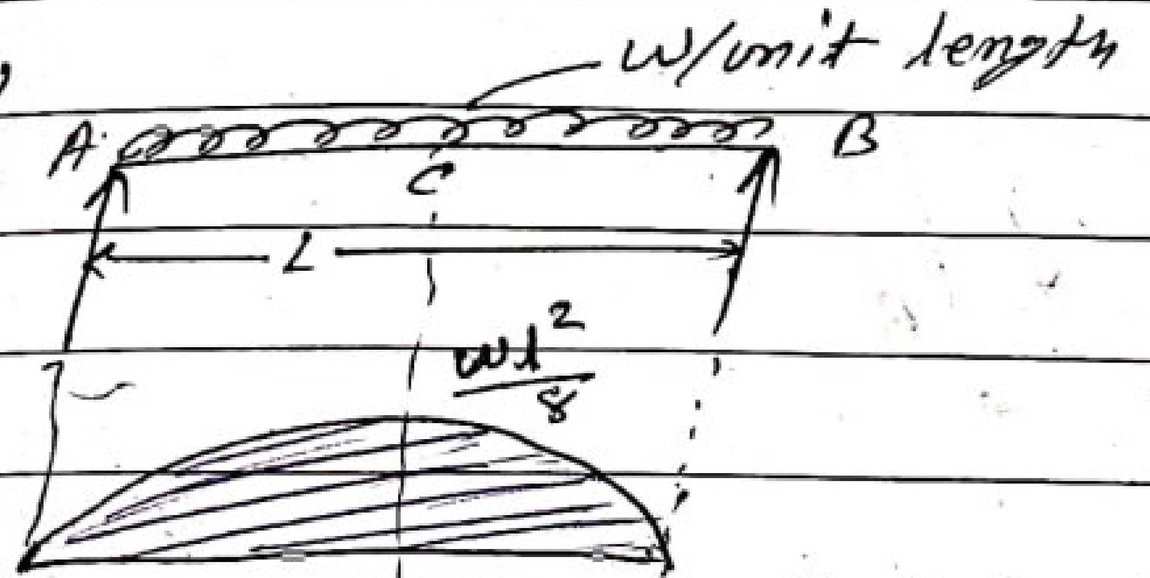
* Practical application of Flexural formula

(i)



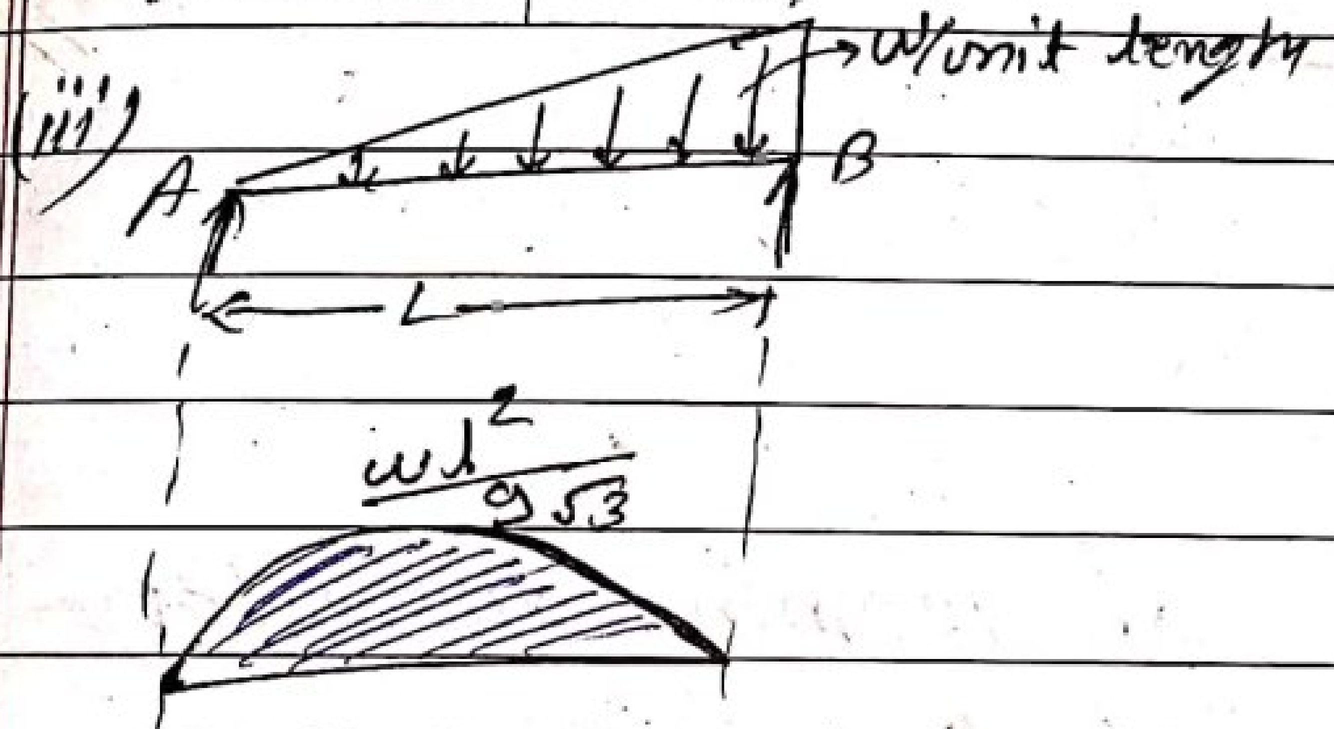
$$M_{max} = \frac{wl}{4} \text{ (Sagging)}$$

(ii)



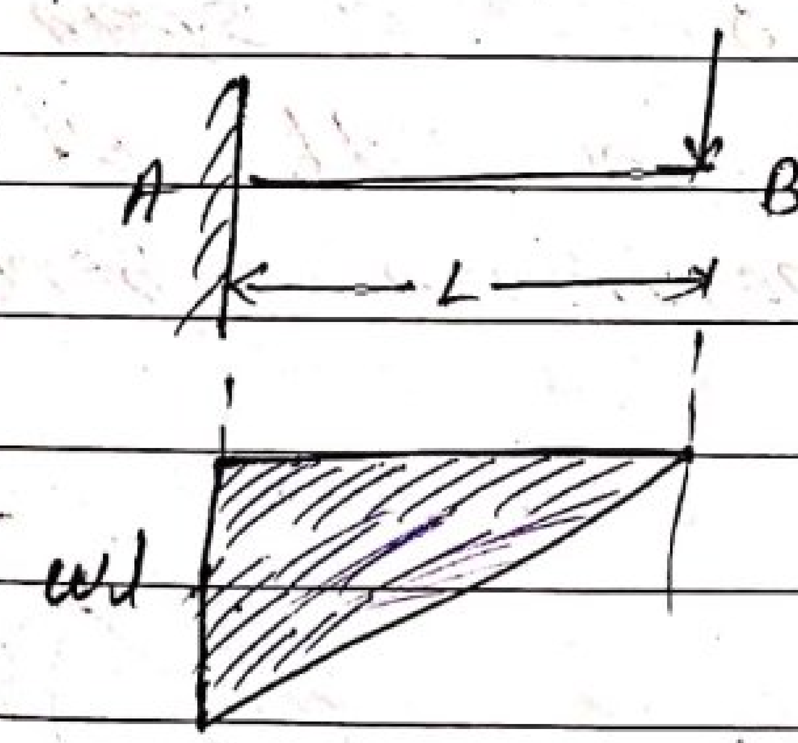
$$M_{max} = \frac{wl^2}{8} \text{ at point } c$$
(Sagging)

(iii)



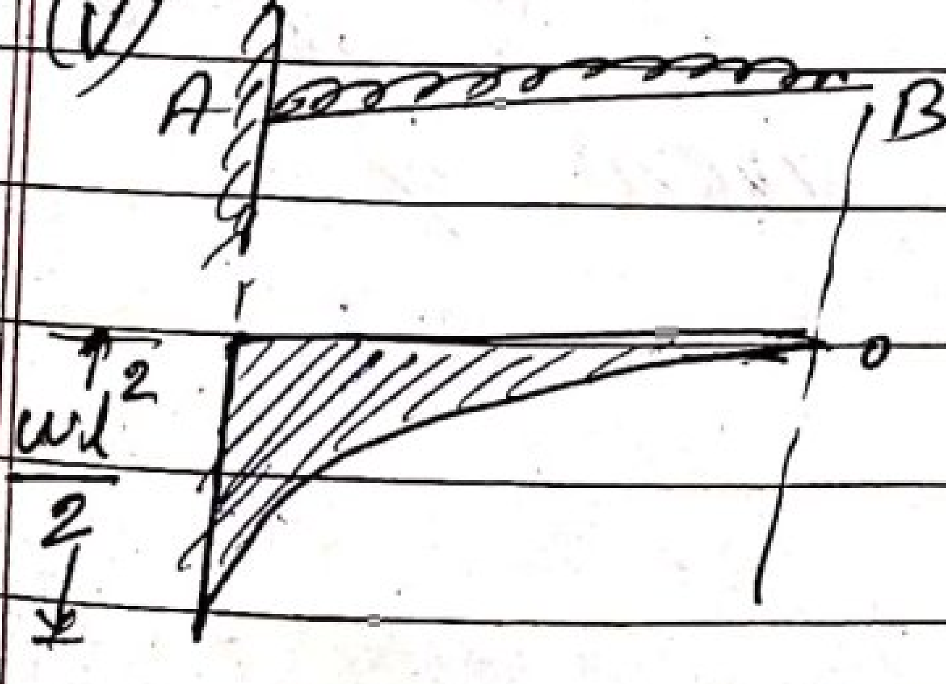
$$M_{max} = \frac{wl^2}{9\sqrt{3}}$$

(iv)



$$M_{max} = -wl$$
$$= wl \text{ (Hogging)}$$

(v)



$$M_{max} = \frac{-wl^2}{2}$$
$$= \frac{wl^2}{2} \text{ (Hogging)}$$

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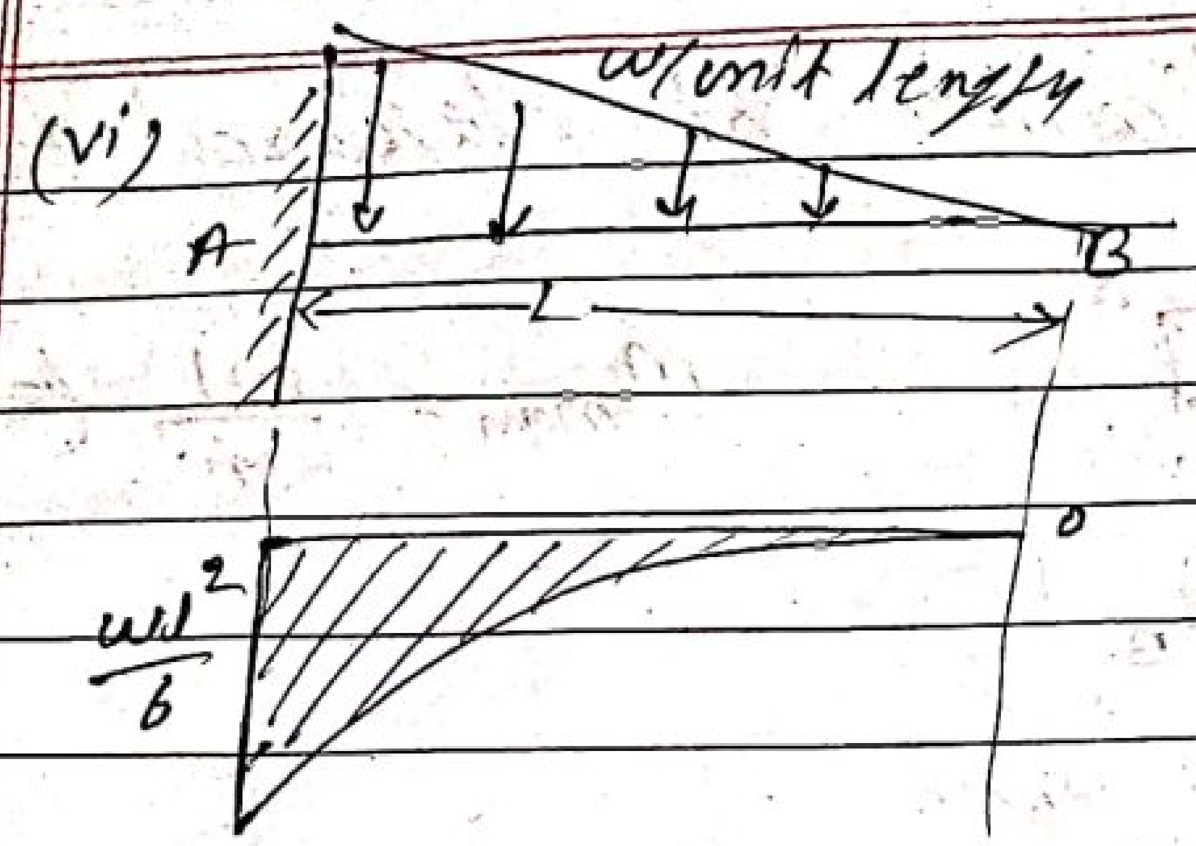
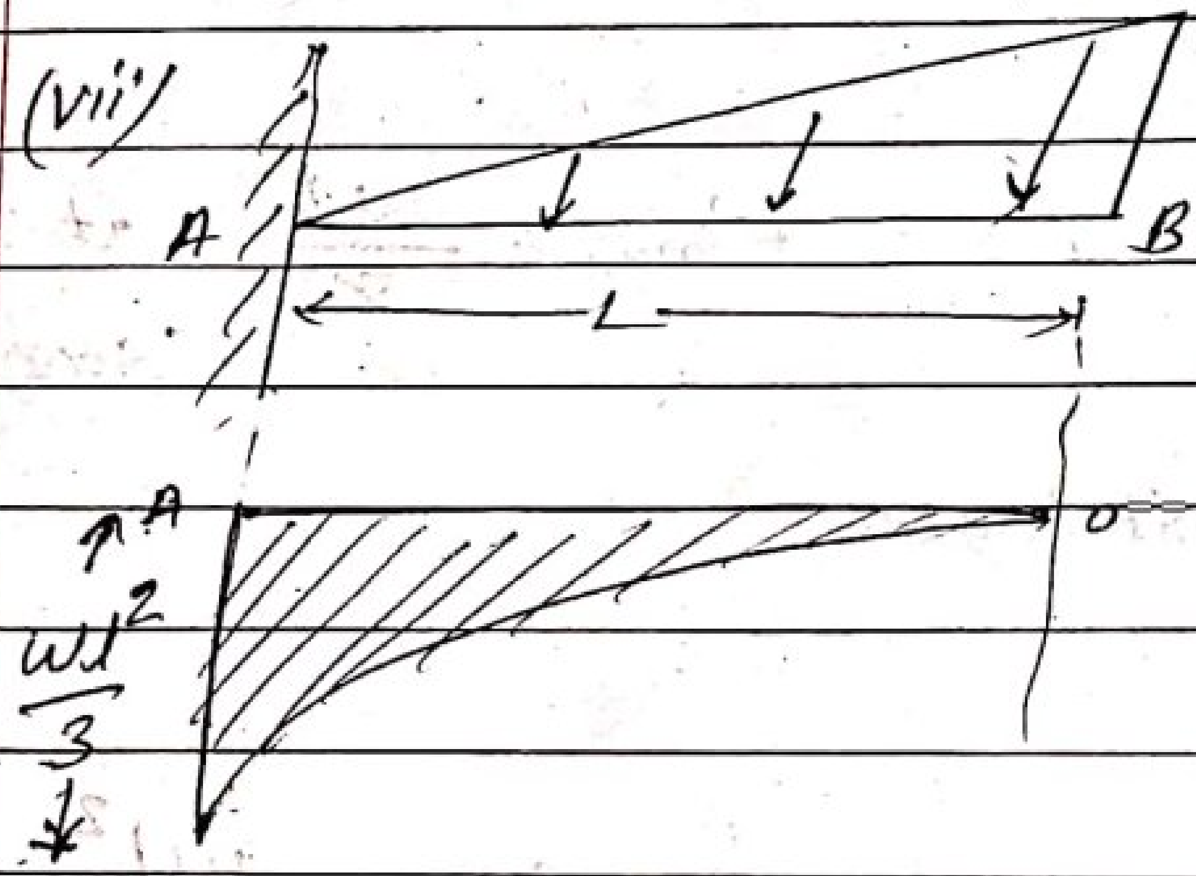
(vi)



$M_{max} = \frac{wl^2}{6}$ (Hogging)

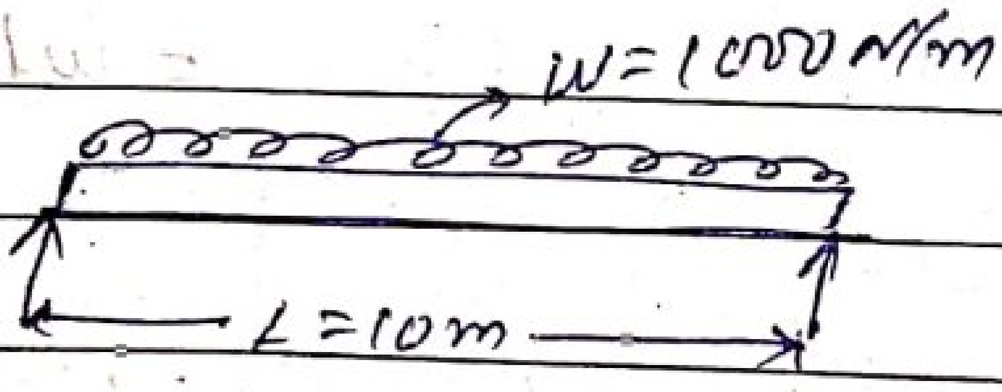
(vii)

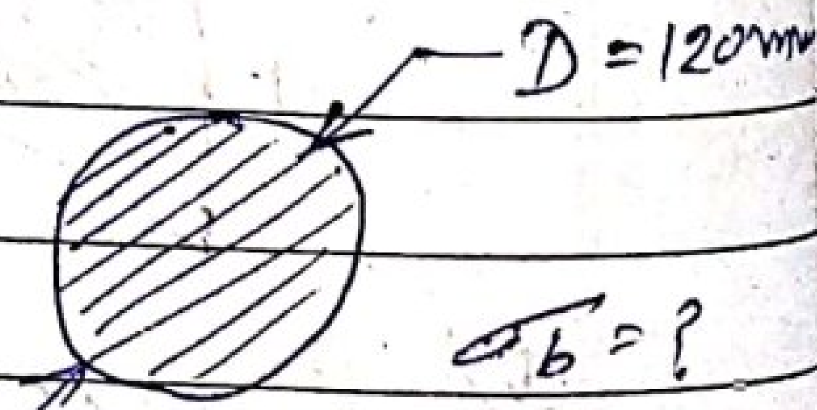


$M_{max} = \frac{wl^2}{3}$ (Hogging)

Q circular beam of 120mm diameter is simply supported over a span of 10m and carries a udl of 1000 N/m. Find the maximum bending stress produced.

Sol^n:-





∵ The Flexural Eq^n is given by

$$\left[\frac{M}{I} = \frac{\sigma_b}{Y} = \frac{E}{R}\right]$$

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$\therefore \dfrac{M}{I} = \dfrac{\sigma_b}{y}$

$$\therefore \boxed{\sigma_b = \frac{M}{I} \cdot y} \quad \text{——— (1)}$$

$\because$ B.M. for give beam is as

$$M = \frac{WL^2}{8}$$

$$= \frac{1000 \times (10)^2}{8} = 12.5 \times 10^3 \ \text{N-m}$$

$$= 12.5 \times 10^6 \ \text{N-mm}$$

$\because$ M.O.I for circular section is given by

$$I = \frac{\pi}{64} D^4$$

$$= \frac{\pi}{64} (120)^4 = 10.18 \times 10^6 \ \text{mm}^4$$

$\because$ Distance of layer subjected to max bending stress from N.A.

$$y = \frac{D}{2} \qquad \therefore y = \frac{120}{2} = 60 \ \text{mm}$$

put All values in Eq^n (1)

$$\therefore \sigma_b = \frac{12.5 \times 10^6}{10.18 \times 10^6} \times 60$$

$$= 73.68 \ \text{N/mm}^2$$

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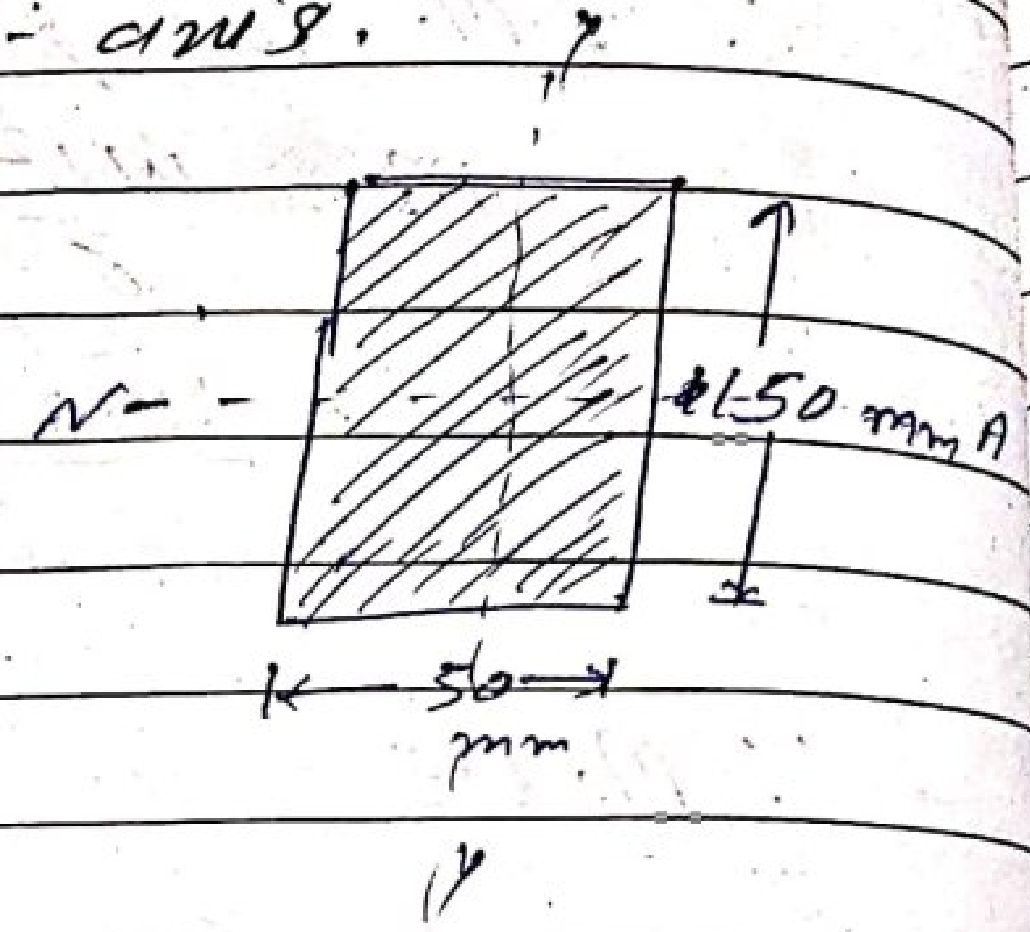
Q Determine the maximum bending stress developed in a beam of rectangular cross section (50mm x 150mm) when a bending moment of 600 N·m is applied about x-x axis.

Solⁿ:- given data

$\sigma_b = ?$

$M = 600$ N·m



∵ The flexural formula is given as

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

$$\therefore \boxed{\sigma_b = \frac{M}{I} y} \quad \text{—— (1)}$$

∵ M.O.I about x-x axis is given by

$$I_{xx} = \frac{bd^3}{12}$$

$$= \frac{50 \times (150)^3}{12} = 14.06 \times 10^3 \text{ mm}^4$$

Now ∵ Distance of layer having maximum stress from N.A

$$y = \frac{d}{2} = \frac{150}{2} = 75 \text{mm}$$

put all values in Eqⁿ (1)

$$\therefore \sigma_b = \frac{600}{14.06 \times 10^3} \times 75 = 3.2 \text{ N/mm}^2$$

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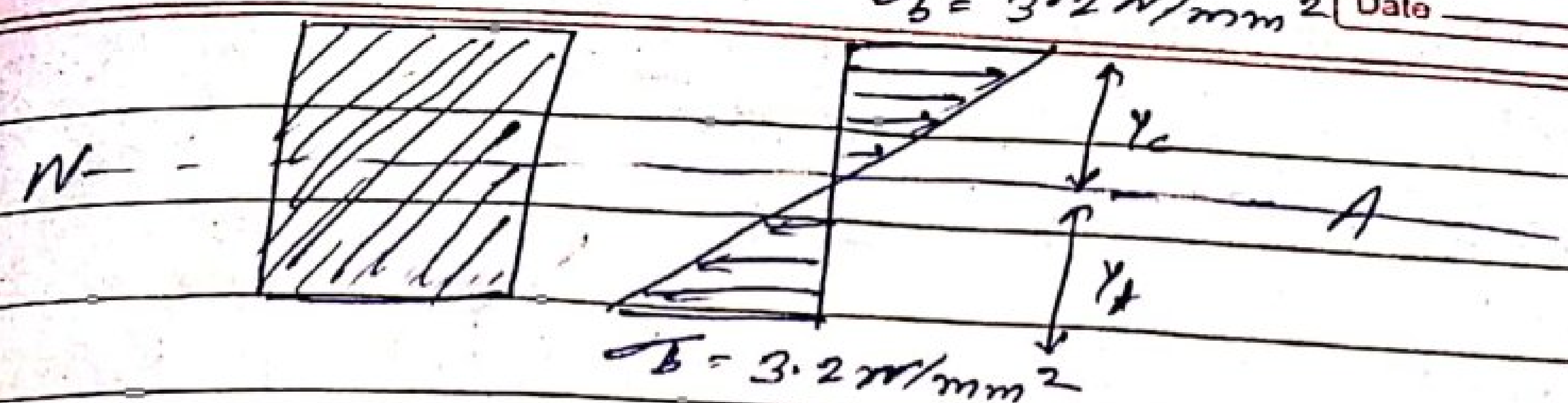
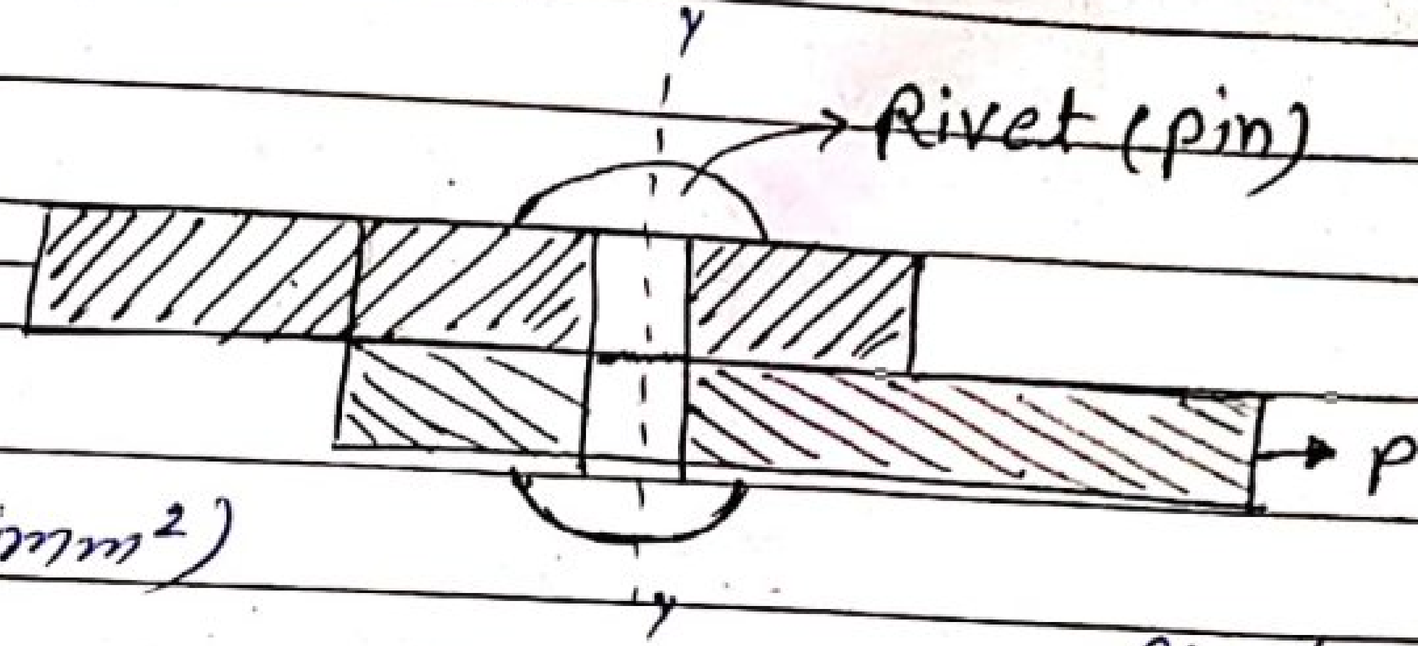
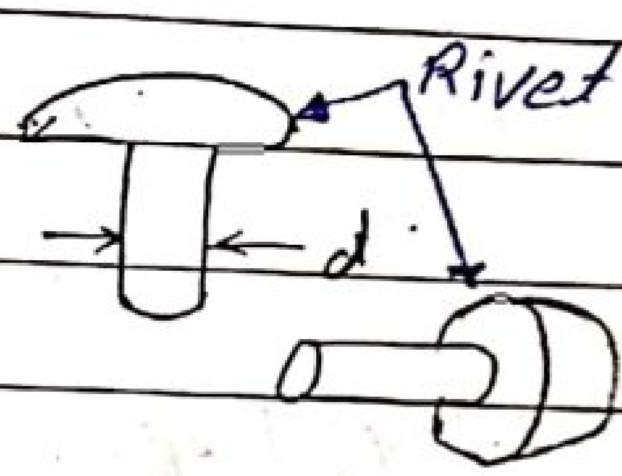


shear stresses



Let
d = diameter of rivet
A = Area of the rivet (mm^2)

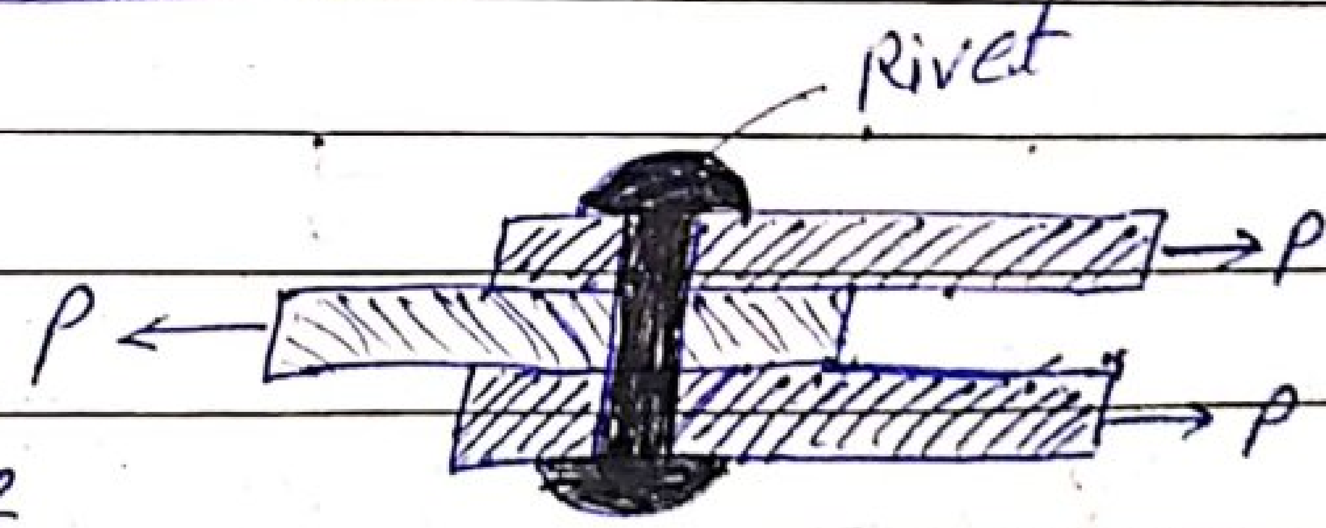


⇒ it is ~~the~~ defined as the ratio of shear force to the cross-section area
shear stress is denoted by 'τ' or f_s

$$\text{shear stress}(\tau) = \frac{\text{Shear force}(P)}{\text{cross-sectional Area}(A)}$$

$$\boxed{\tau = \frac{P}{A} = \frac{P}{\frac{\pi}{4}d^2}}\ N/mm^2$$

(ii) Double shear:-



∴ Resisting area of rivet (A) $= 2 \times \frac{\pi}{4} d^2$

1+1 = 2 double shear

$$\text{shear stress}(\tau) = \frac{\text{shear force}(F)}{\text{cross-section Area}(A)}$$

$$\boxed{\tau = \frac{P}{2\frac{\pi}{4}d^2}}$$

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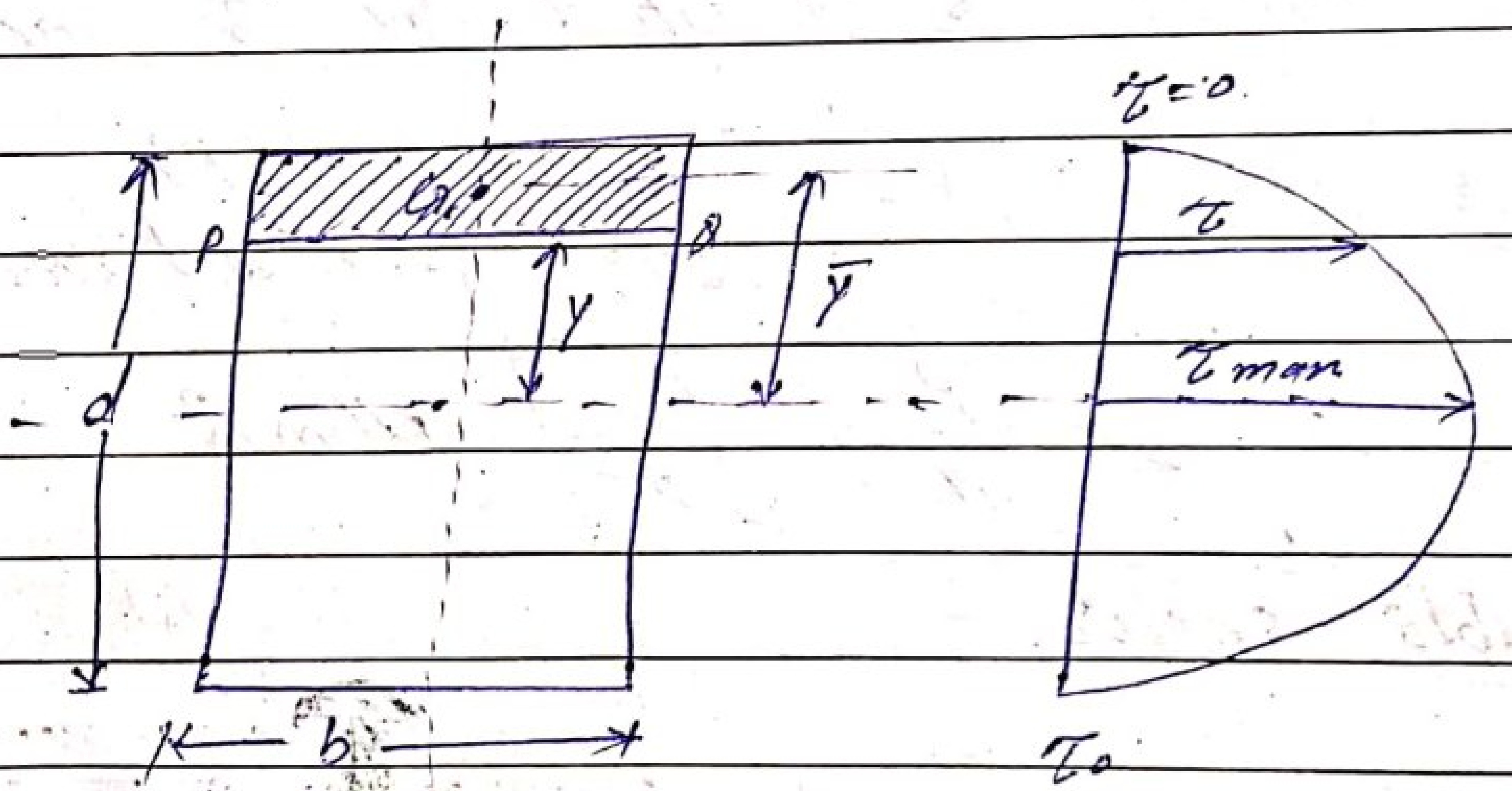
Note:-
shear stress (τ) = $\frac{F A \bar{Y}}{I \cdot b}$

where:-

τ = shear stress
F = shear force
A = Area of section about N.A
$\bar{Y}$ = Distance of centre of gravity of the area ~~about~~ above N.A
I = moment of inertia
b = width of section where shear stress is calculated

(1) Rectangular section of Area (b×d)



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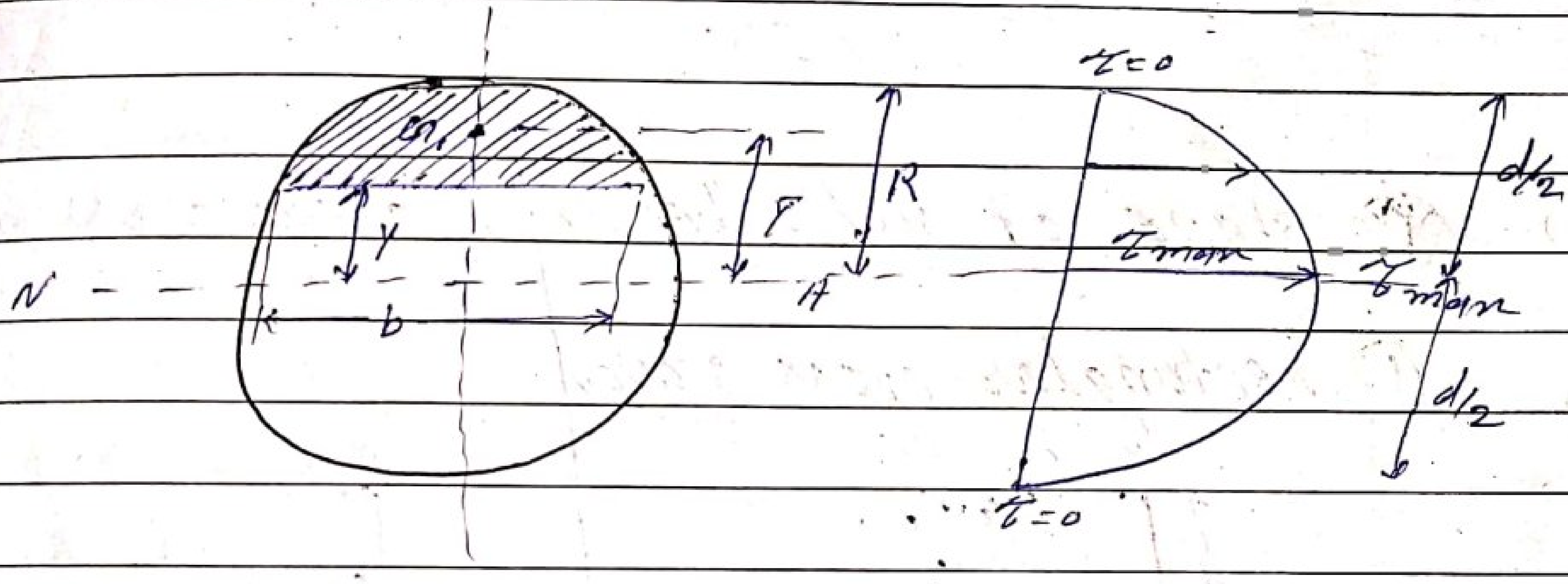
$$\boxed{\tau_{max} = 1.5\ \tau_{average}}$$

$$\boxed{\tau_{max} = 1.5\ \frac{P}{A}}$$

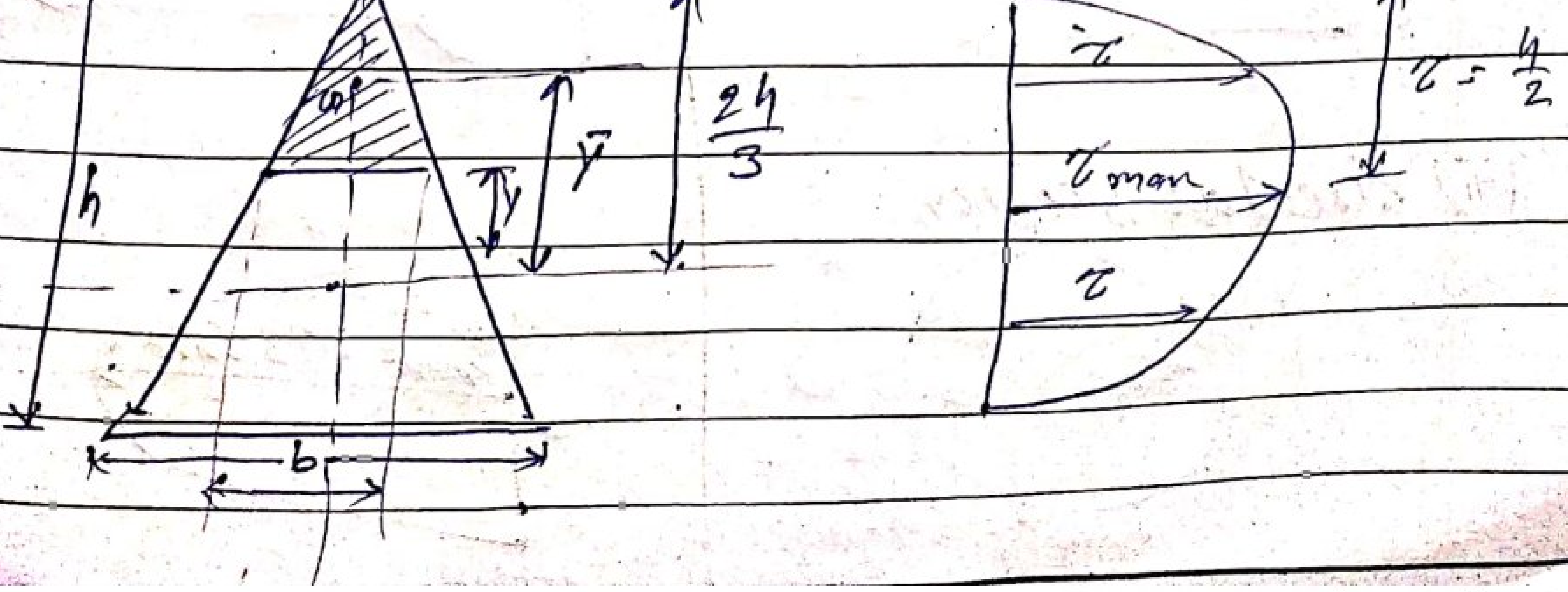
(ii) shear ~~d~~ stress distribution for a circular section.



$$\left[\tau_{ave} = \frac{F}{A}\right]$$

$$\boxed{\tau_{max} = \frac{4}{3}\ \tau_{ave}}$$

(iii) Isosceles ~~triangle~~ triangular section:-



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$\boxed{\tau_{max} = 1.5\ \tau_{avg}}$

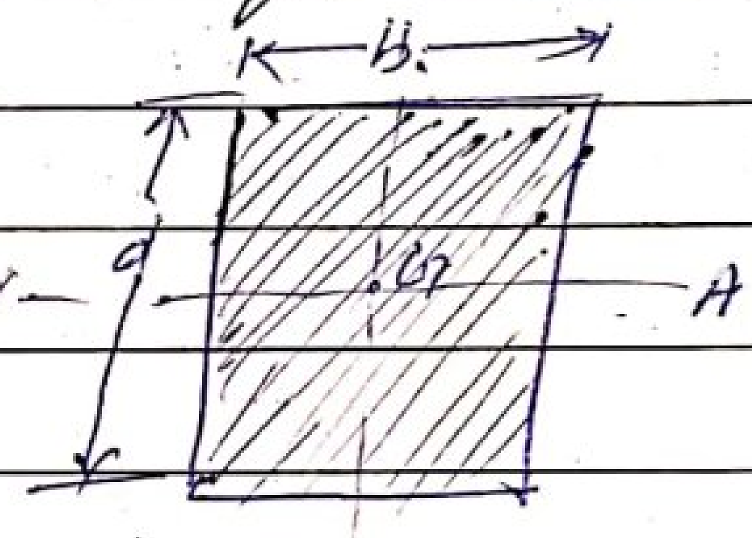
stress Distribution in I-section

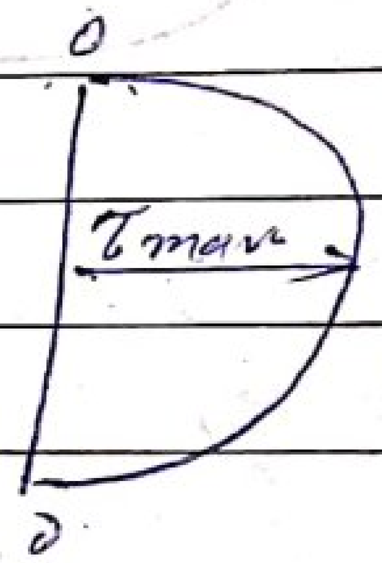
$$\tau_{flange} = \frac{F}{8I}(D^2-d^2)$$

$$\tau_{web} = \frac{F\ B}{8Ib}(D^2-d^2)$$

shape of section beam	shear stress Distribu…

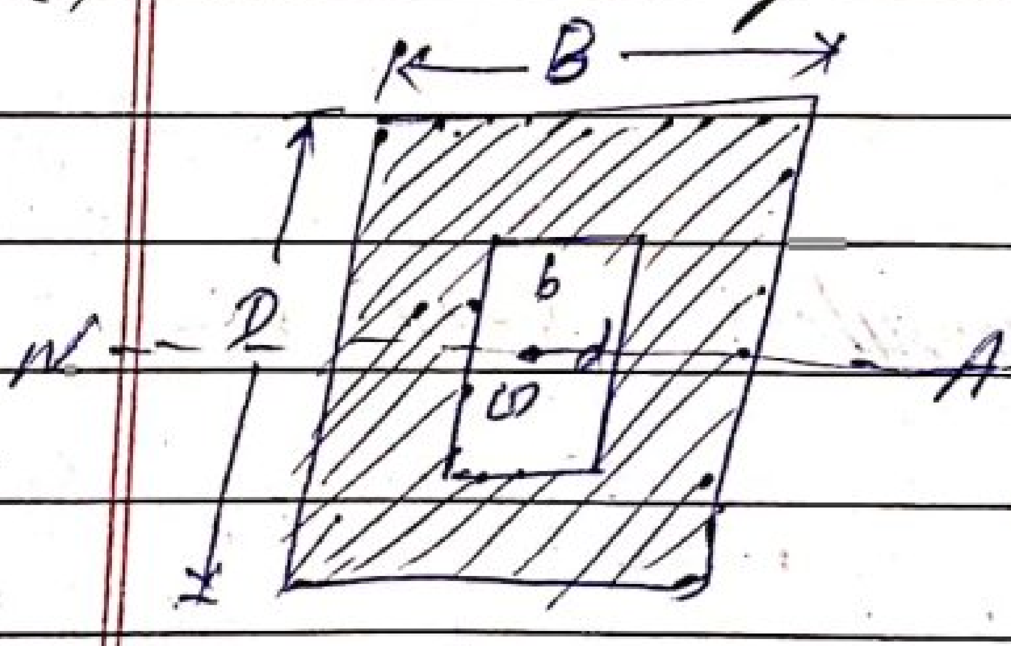
(1) Rectangular cross-section

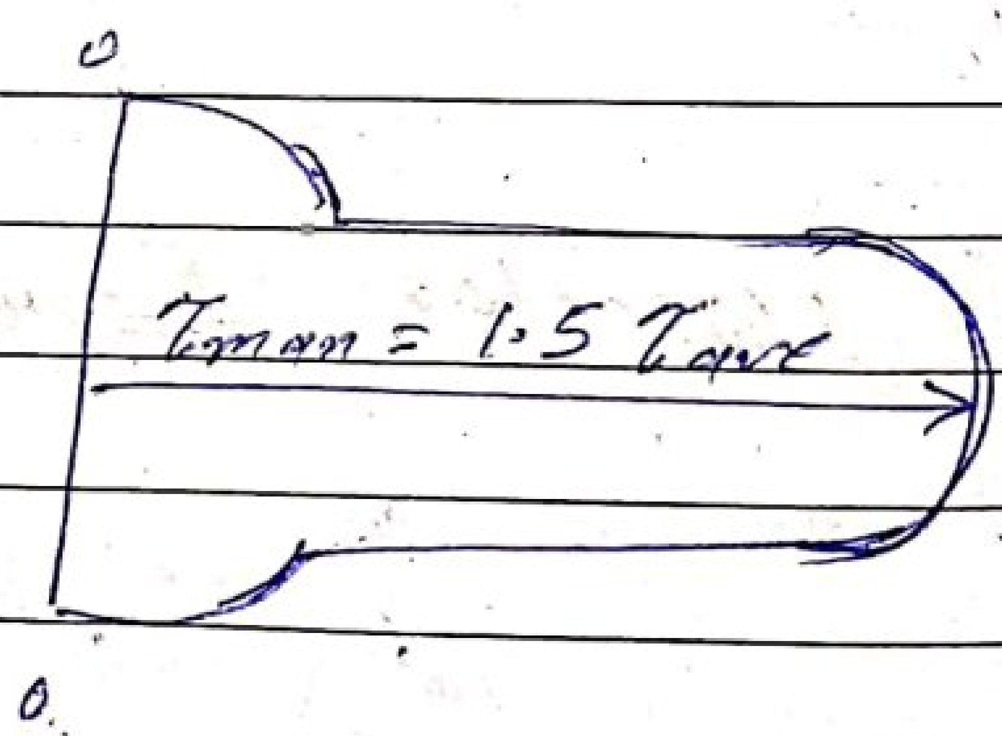




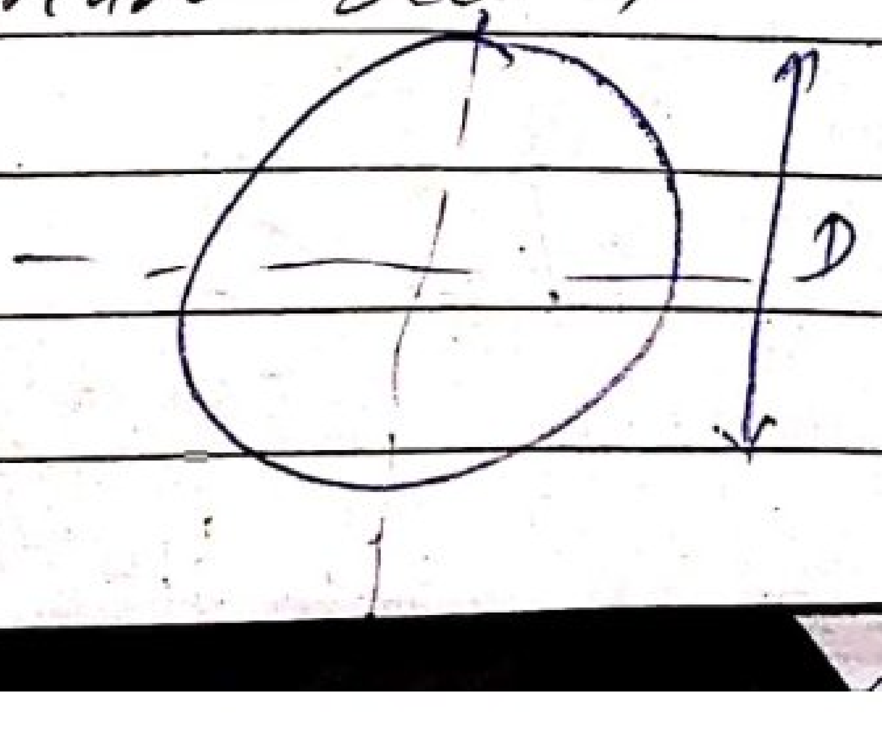
$\boxed{\tau_{max} = 1.5\,\tau_{av…}}$

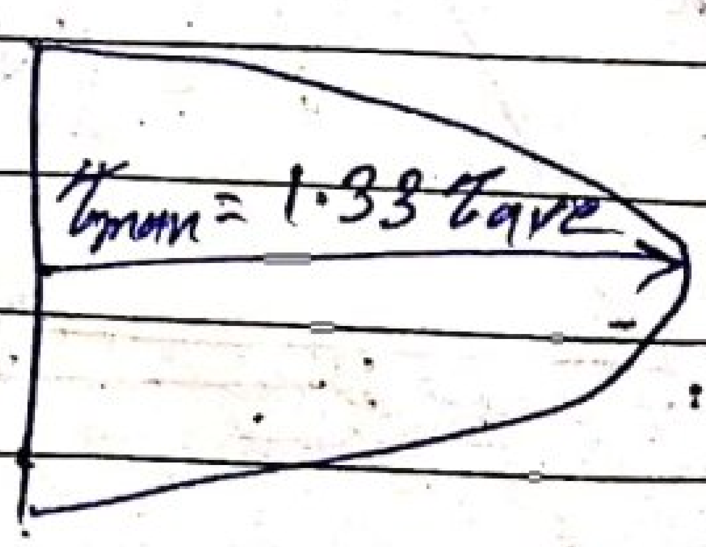
(ii) Hollow Rectangular





(iii) circular section





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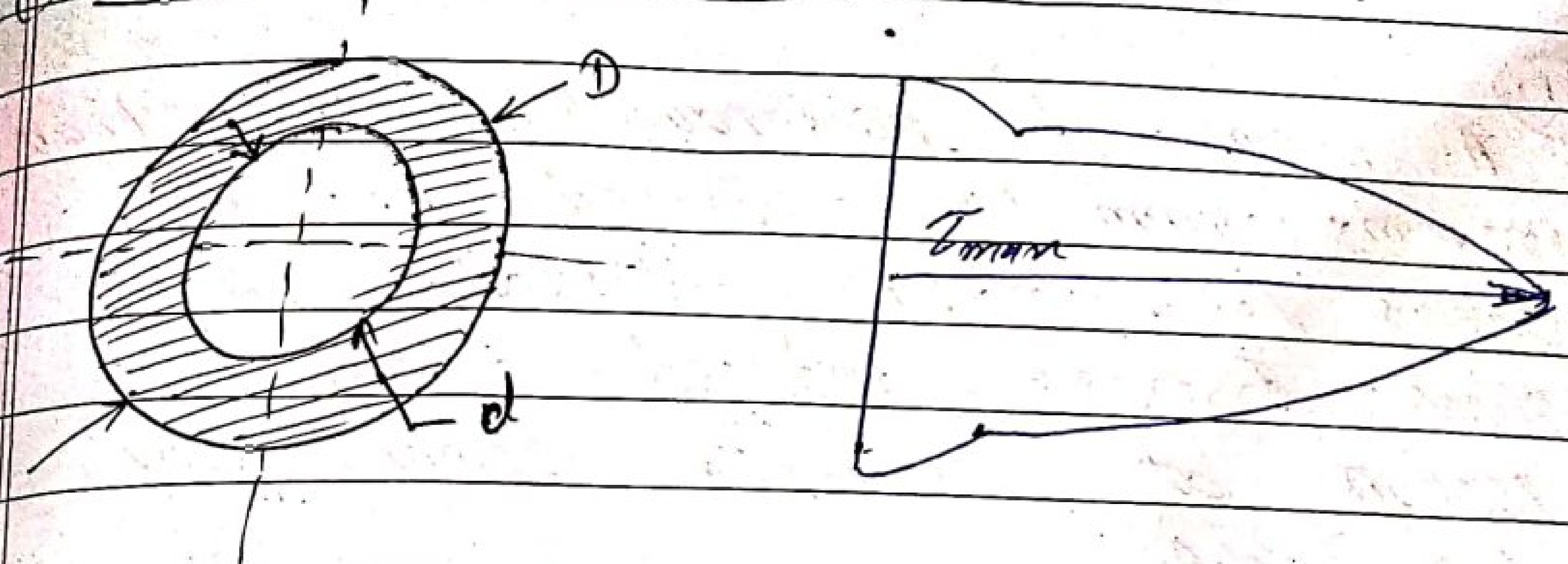
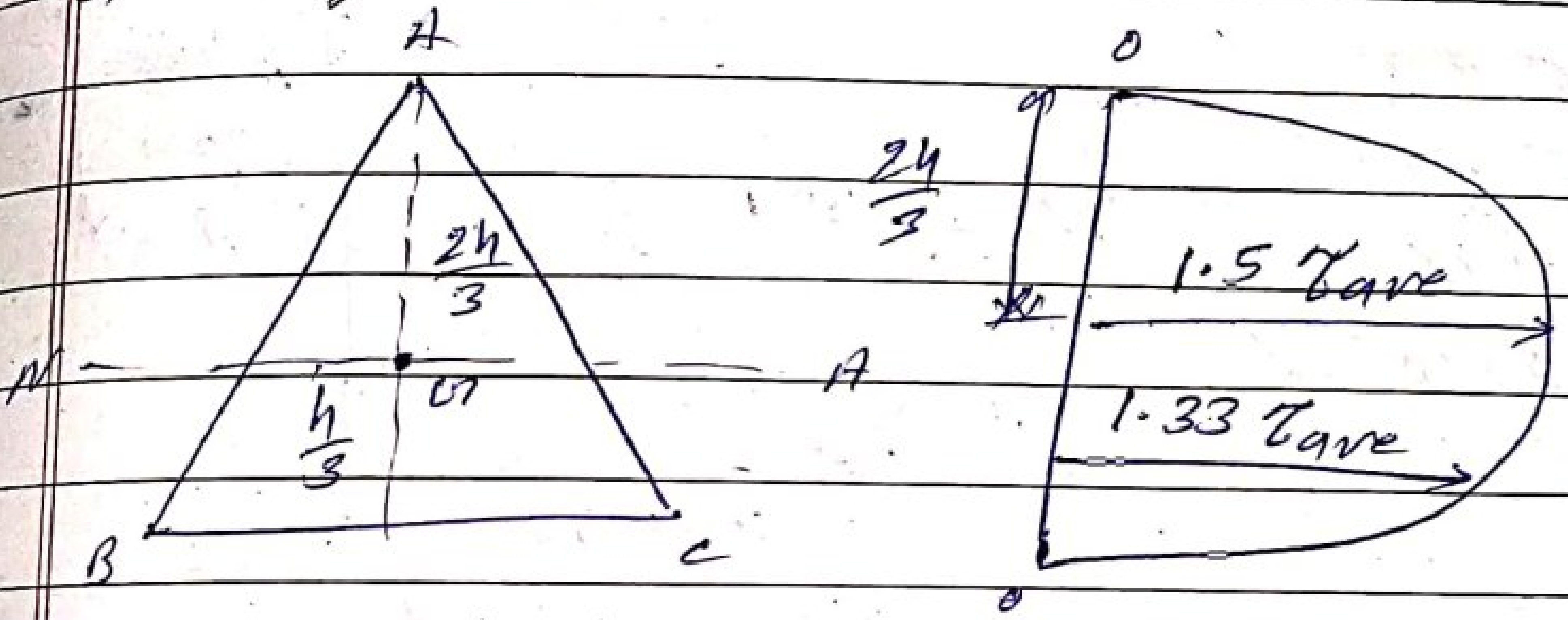
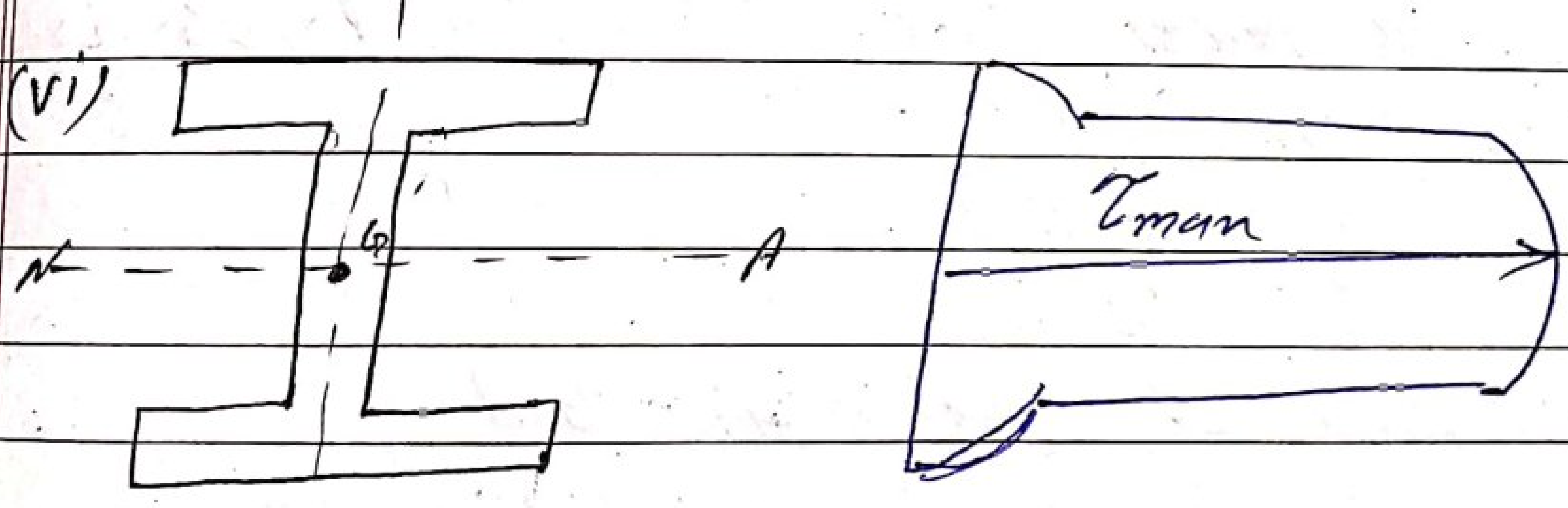
(iv) Hollow circular section



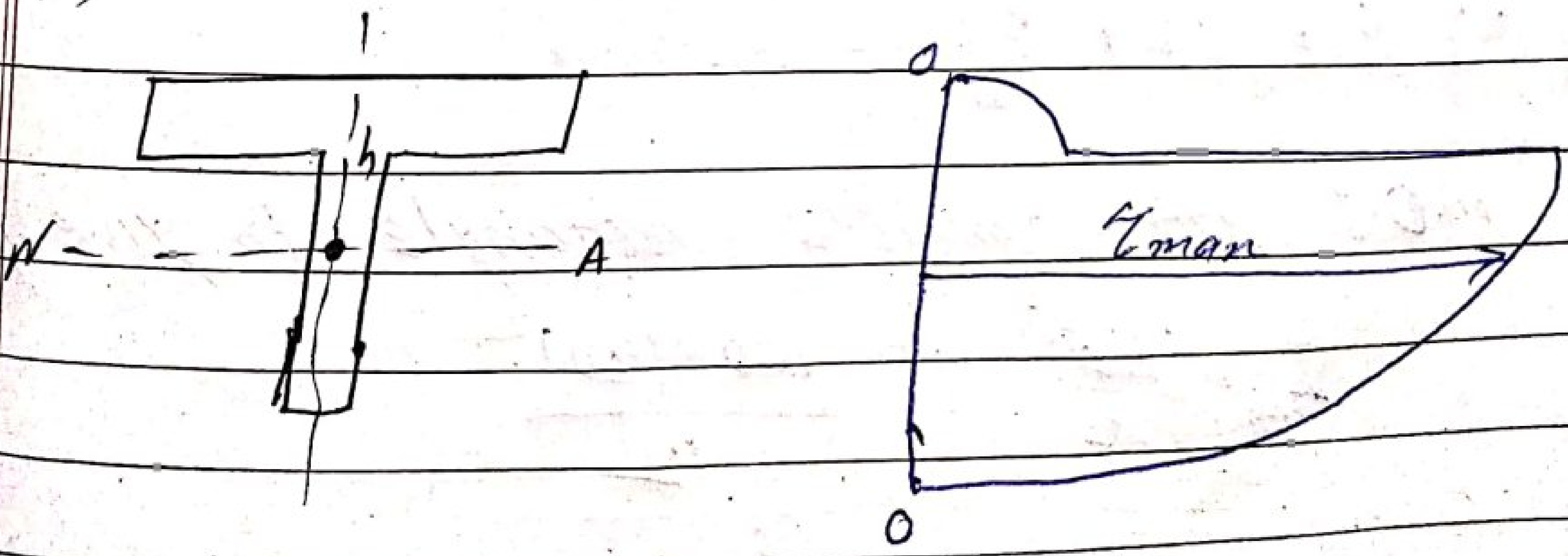
(v) Triangular section



(vi)



(vii) T-section



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Numerical

(1) A beam of Rectangular cross-section 100mm × 100mm is subjected to a shear force 30 KN. calculate the shear stress induced across the section at a layer 20mm away from N.A.

Soln:- given data

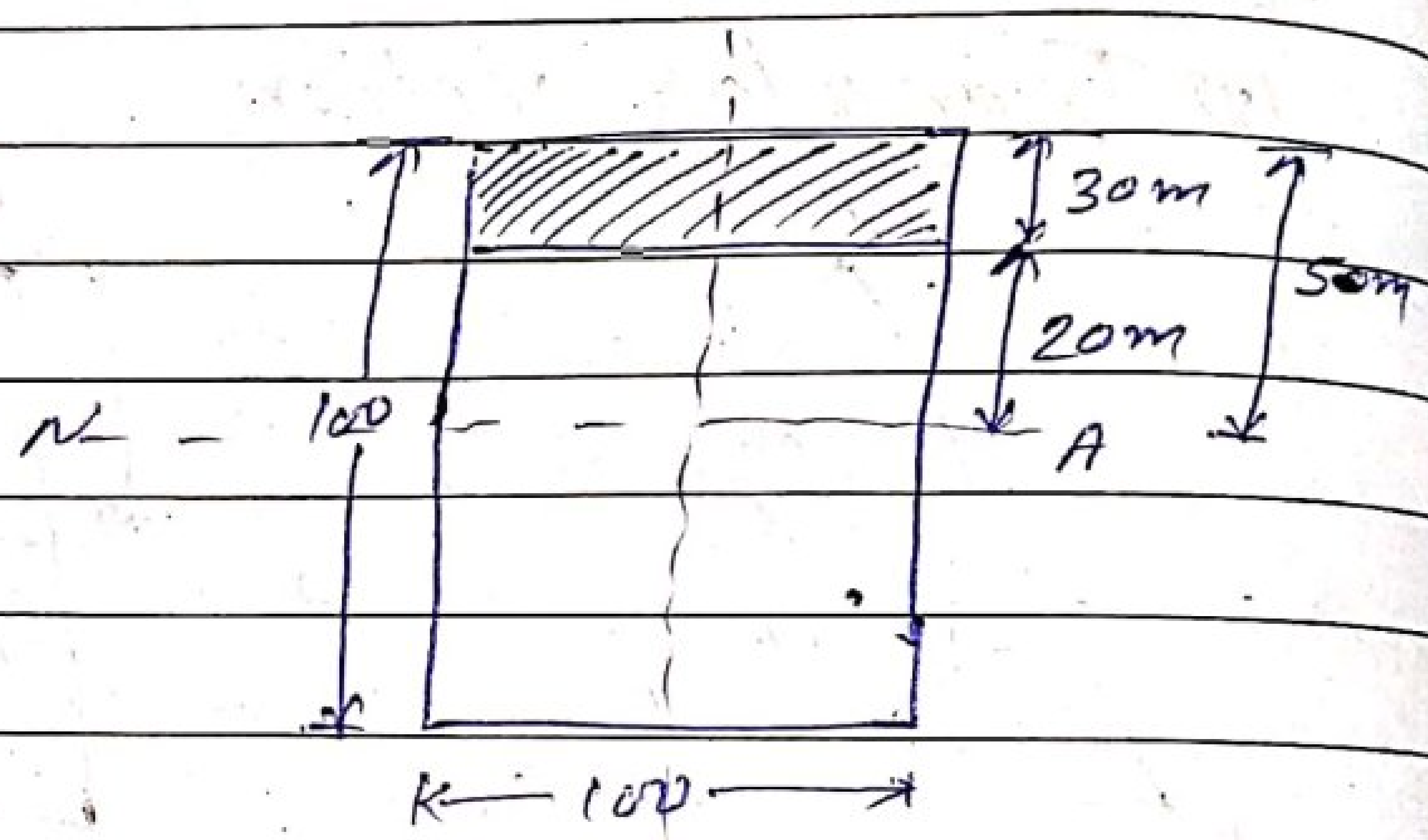
$b = 100 mm$

$d = 100 mm$

$F = 30 KN$

$= 30 \times 10^3 N$

$\tau = ?$



∵ shear stress at any give layer is written as

$$\tau = \frac{F \cdot A \cdot \bar{y}}{I \cdot b}$$

∴ Area of section (A) = b×d

$= 100 \times 30$

$= 3000 mm^2$

∵ $\bar{y} = 20 + \frac{30}{2} = 35 mm$

∵ M.O.I for complete rectangle is give by

$$I_{N.A} = \frac{bd^3}{12} = \frac{100 \times (100)^2}{12}$$

$$\boxed{I_{N.A} = 8.33 \times 10^6 mm^4}$$

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put all value in Eqⁿ ①

$$\therefore \tau = \frac{30\times10^3 \times 3\times10^3 \times 35}{8.33\times10^6 \times 100}$$

$$\therefore \boxed{\tau_{AB} = 3.78\ N/mm^2}$$

∵ Average shear stress is given by

$$\tau_{ave} = \frac{F}{A}$$

$$\therefore \tau_{ave} = \frac{30\times10^3}{100\times100}$$

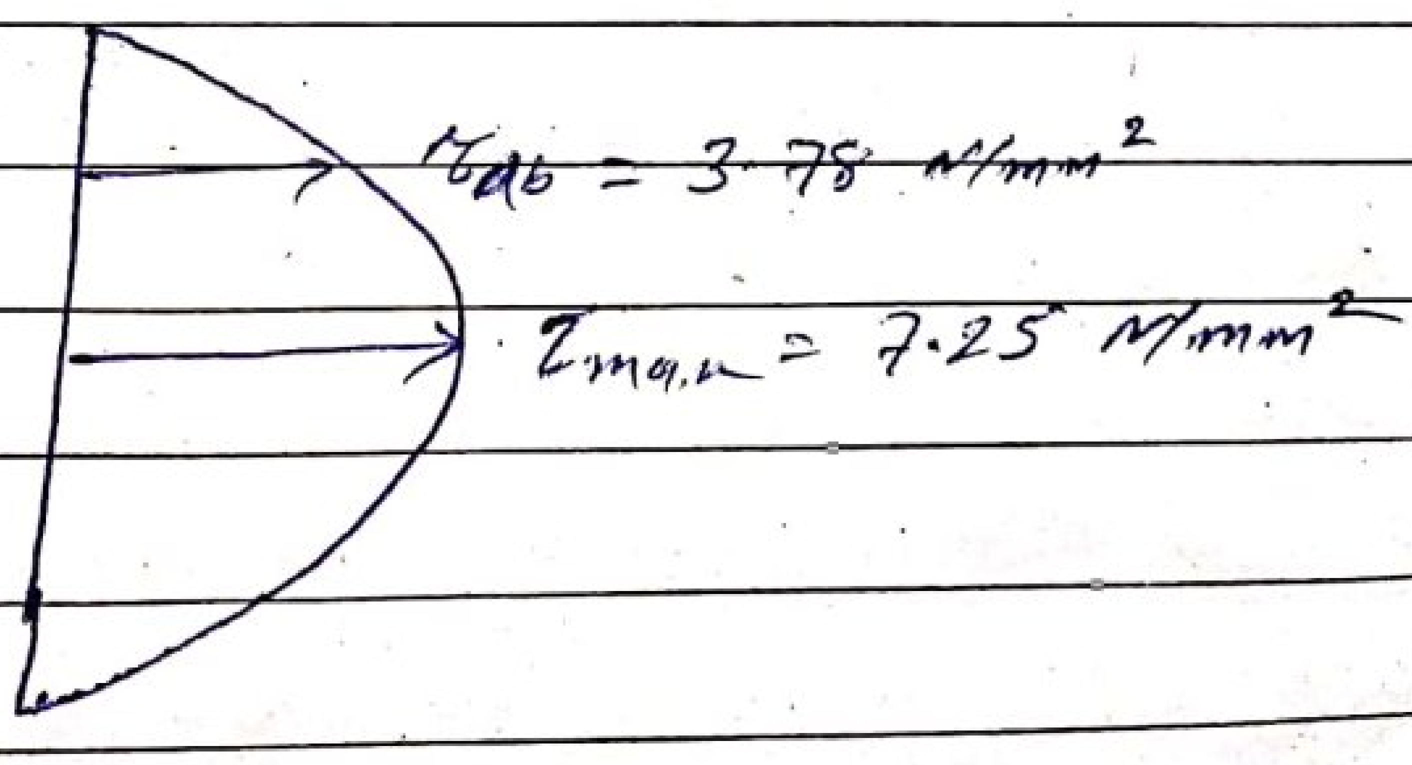
$$\therefore \tau_{ave} = 1.5\ N/mm^2$$

∵ Max shear stress is given by

$$\tau_{max} = 1.5 \times \tau_{ave}$$

$$= 1.5 \times 4.5$$

$$= 7.25\ N/mm^2$$



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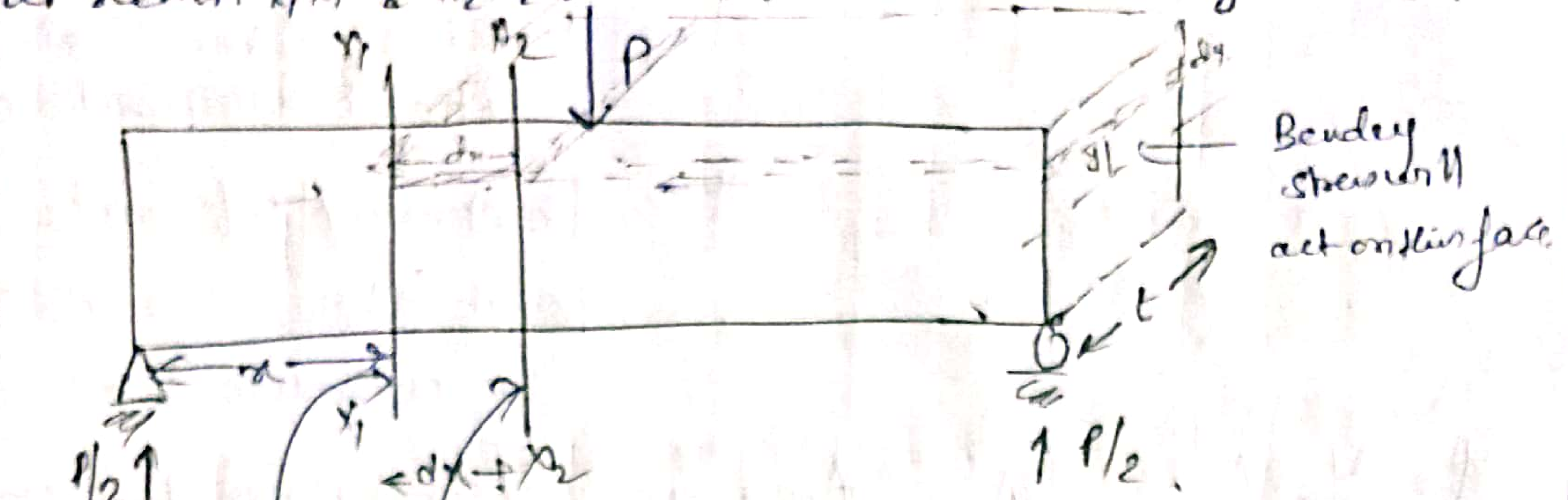
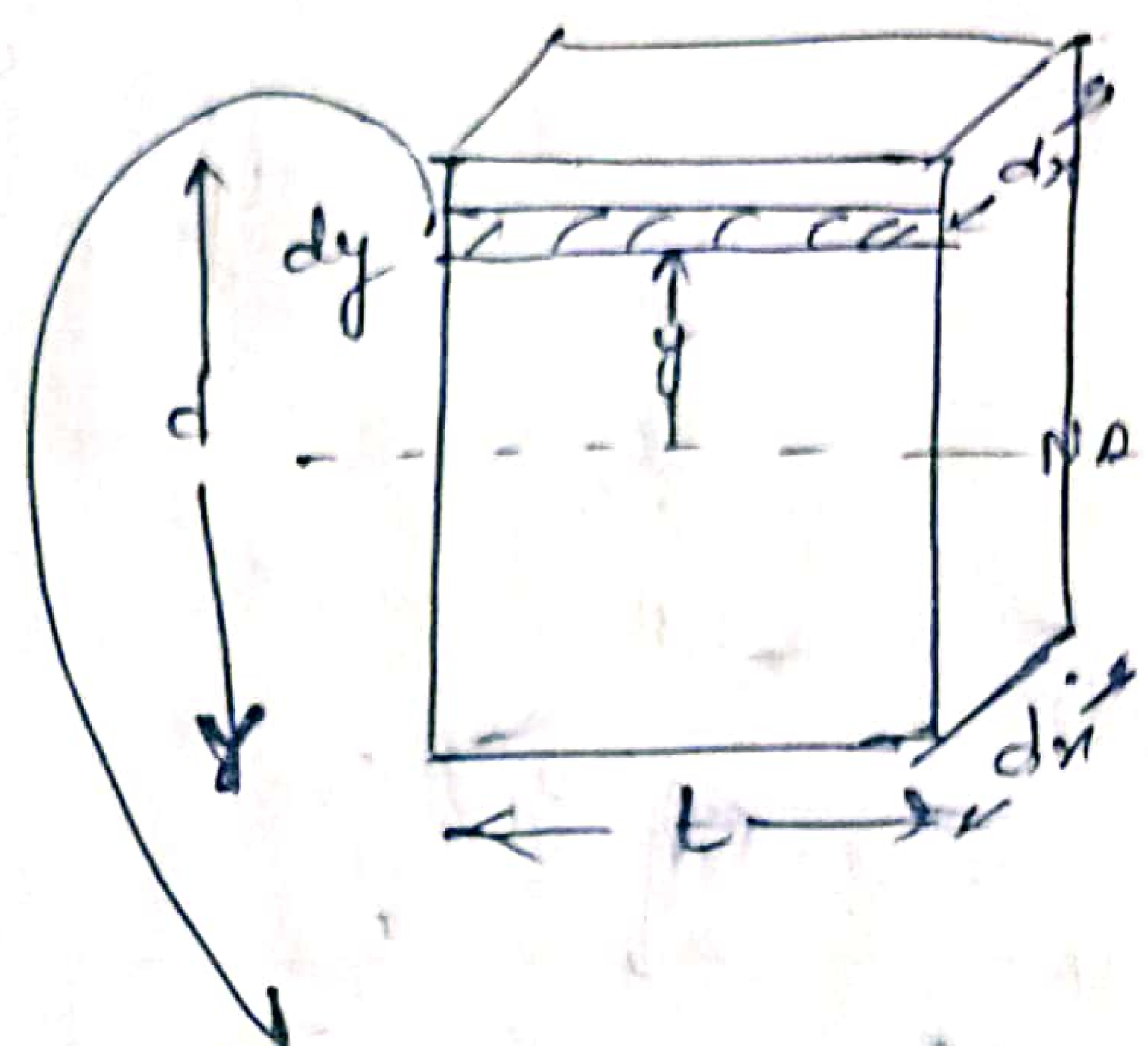
* **Direct shear stress:** Force developed on section x_1x_1 & x_2x_2 is due to bending stress which acts perpendicular to the surface and this bending stress is differ at section x_1x_1 & x_2x_2 due variation in bending moment.



$\frac{P}{2}x = M$ $\qquad$ $\frac{P}{2}(x+dx) = M+dm$

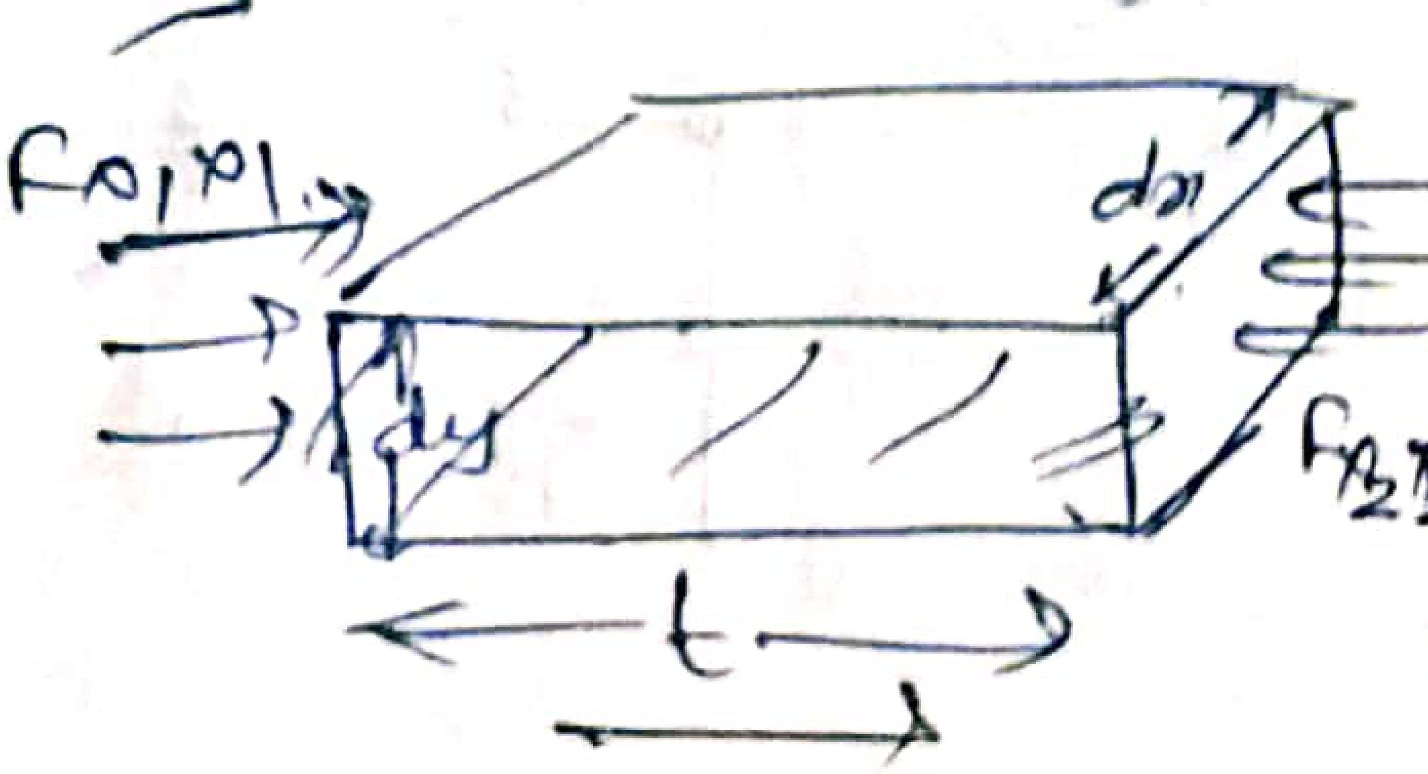


$\sigma = \frac{My}{I}$

$\sigma_{x_1x_1} = \frac{My}{I}$

$\sigma_{x_2x_2} = \frac{(M+dM)y}{I}$

$F_{x_1x_1} = \left(\frac{My}{I}\right) t\,dy$



$F_{x_2x_2} = \frac{(M+dM)y}{I}\, t\,dy$

$F' = \tau \cdot t\,dx$

So $F_{x_1x_1} + F' = F_{x_2x_2}$

$$F' = \left[\frac{(M+dM)y}{I} - \frac{My}{I}\right] t\,dy$$

$$\tau \cdot t\,dx = \frac{dM}{I}\, y \cdot t\,dy$$

$$\boxed{\tau = \left(\frac{dM}{dx}\right)\frac{y}{It}(t\,dy) = \frac{V \cdot (\int y\,dA)}{It} = \frac{VQ}{It}}$$

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$$\boxed{\tau = \frac{VQ}{It}}$$

V = Shear force at that section where shear stress is measured.

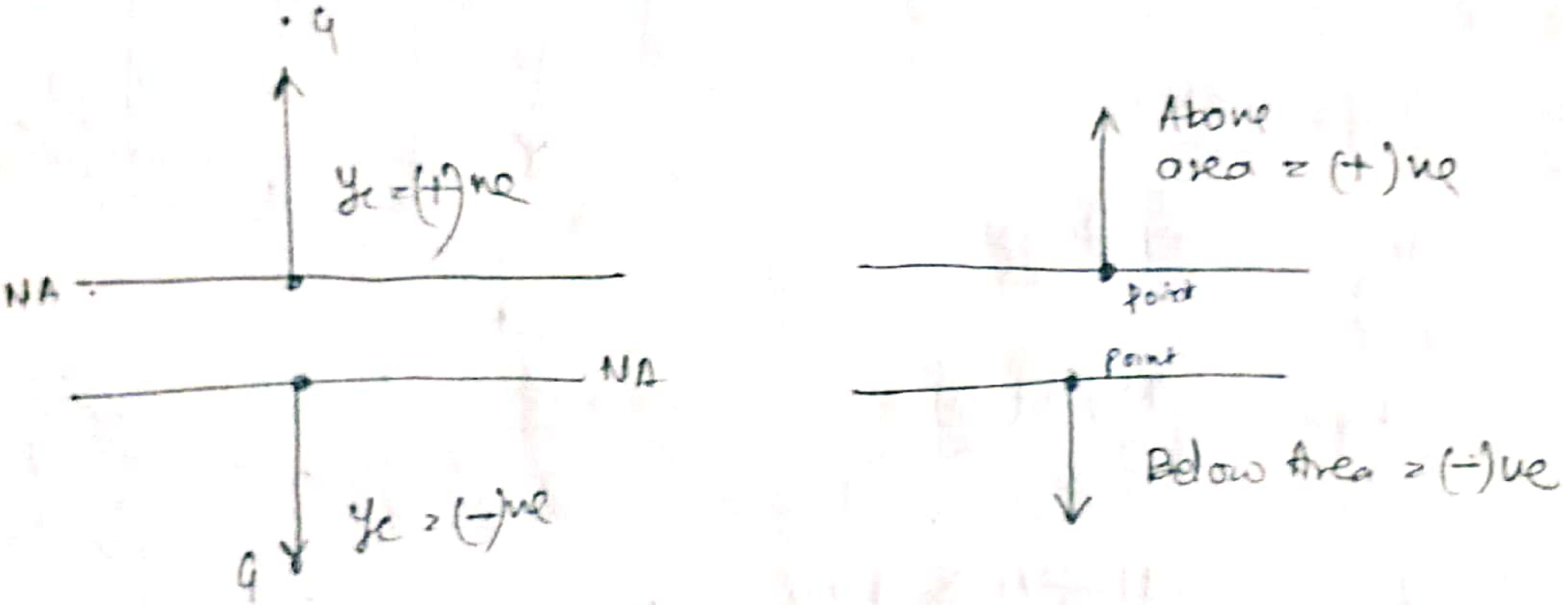
I = M.O.I of Cross-section about neutral axis

t = Thickness of c/s at that point where shear stress is measured.

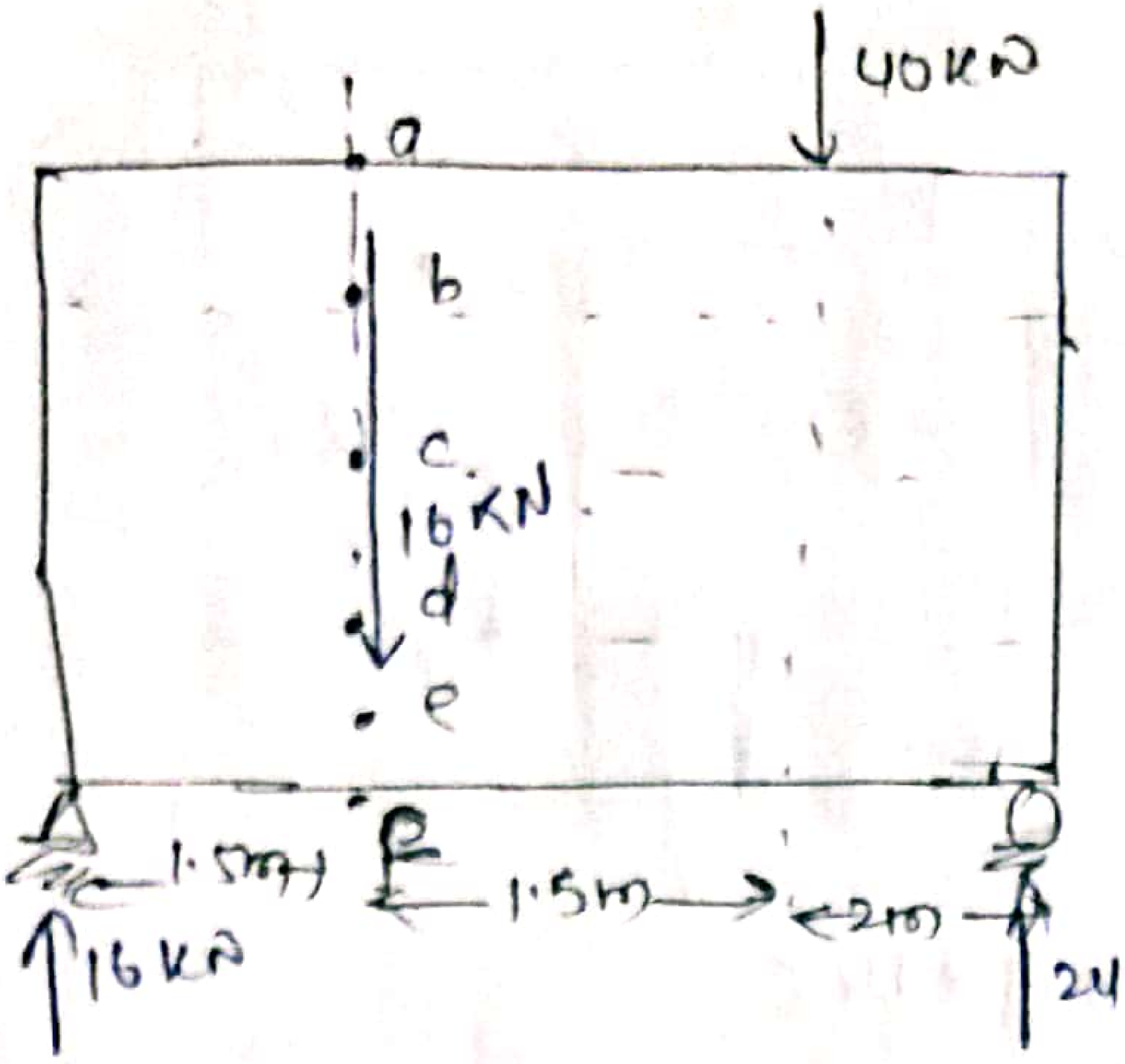
$Q = y_c \cdot A$

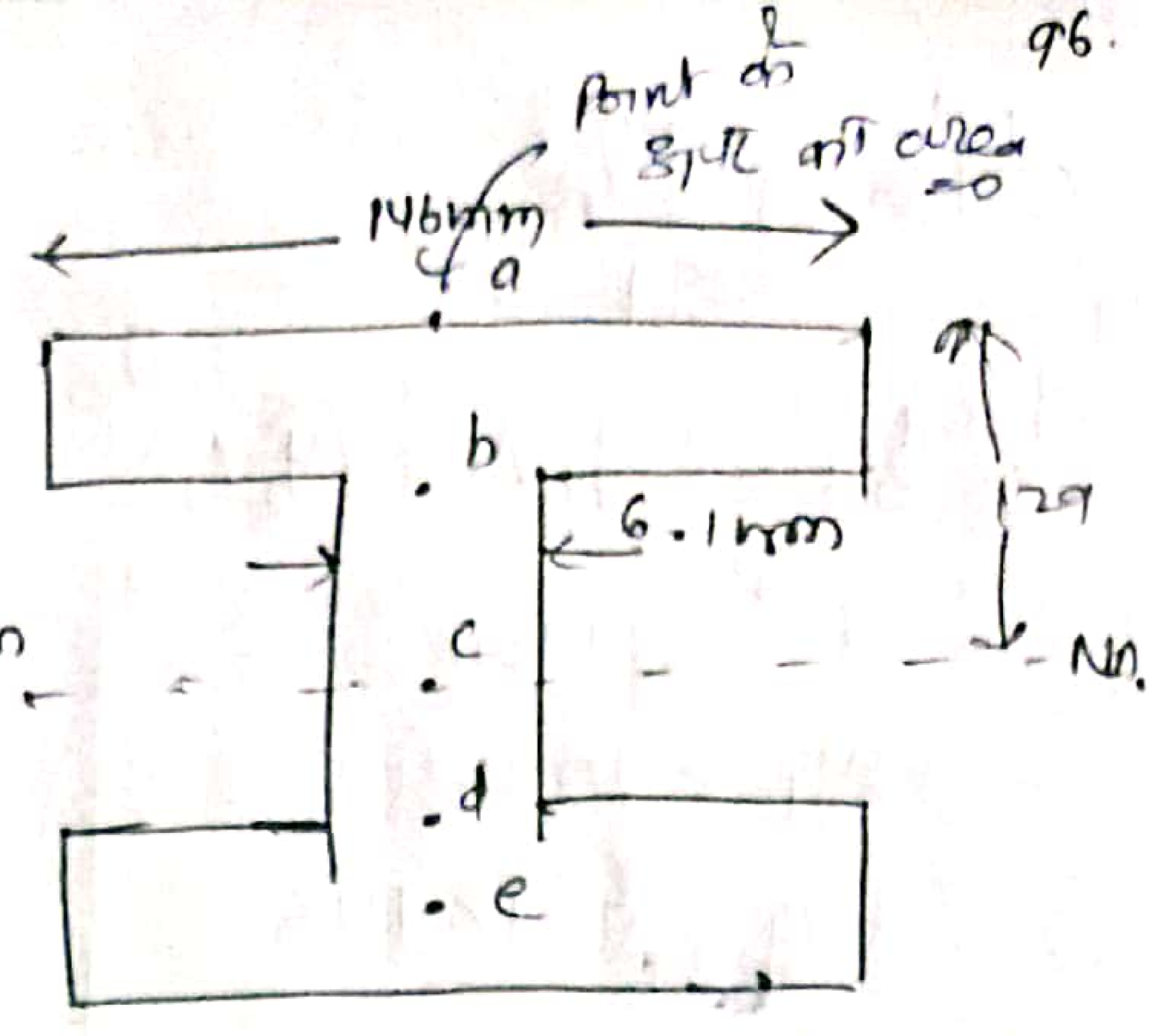
A = Area above (or below) the point where shear stress are measured.

y_c = distance of Centroid of above area (below area) from neutral axis.



Q A wide flanged beam is loaded and supported as shown in fig. Find out the direct shear stress at point A, B, C, D, E. at distance 1.5 m from the left end as shown in fig.



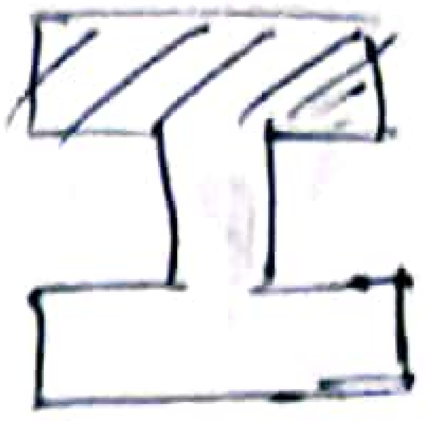


$I_{NA} = 49.1 \times 10^6 \ mm^4$

$\tau_a = \dfrac{VQ_a}{It}$ as $Q_a = y_c A$ & $A = 0$. So $\tau_a = 0$.

$\tau_b = \dfrac{VQ_b}{It}$ = $Q_b = y_b \cdot A$ & $A = 146 \times 9.1 mm$

$y_b = 129 - \dfrac{9.1}{2} = 124.45$



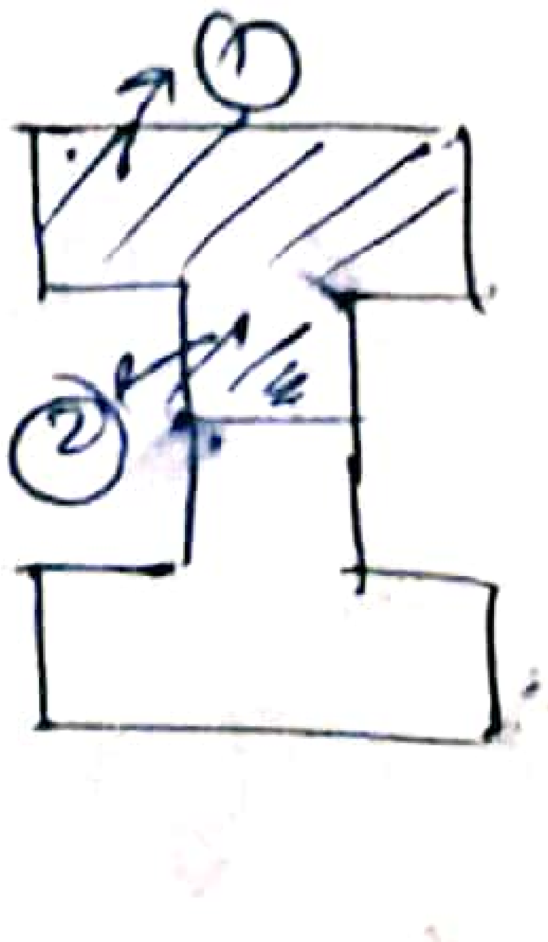
$= \dfrac{16 \times 10^3 \left(129 - \frac{9.1}{2}\right) 146 \times 9.1}{49.1 \times 10^6 \times 6.1 \ mm}$

$= 8.83 \ N/mm^2$

$\tau_c = \dfrac{VQ_c}{It}$ & $Q_c = y_1 A_1 + y_2 A_2$.

$= 124.45 \times 146 \times 9.1$

$+ \dfrac{119.9}{2} \times 6.1 \times 119.9$



$= 209191.1005 \ mm^3$

$= \dfrac{16 \times 10^3 \times 209191.1005}{49.1 \times 10^6 \times 6.1} = 11.175 \ N/mm^2$

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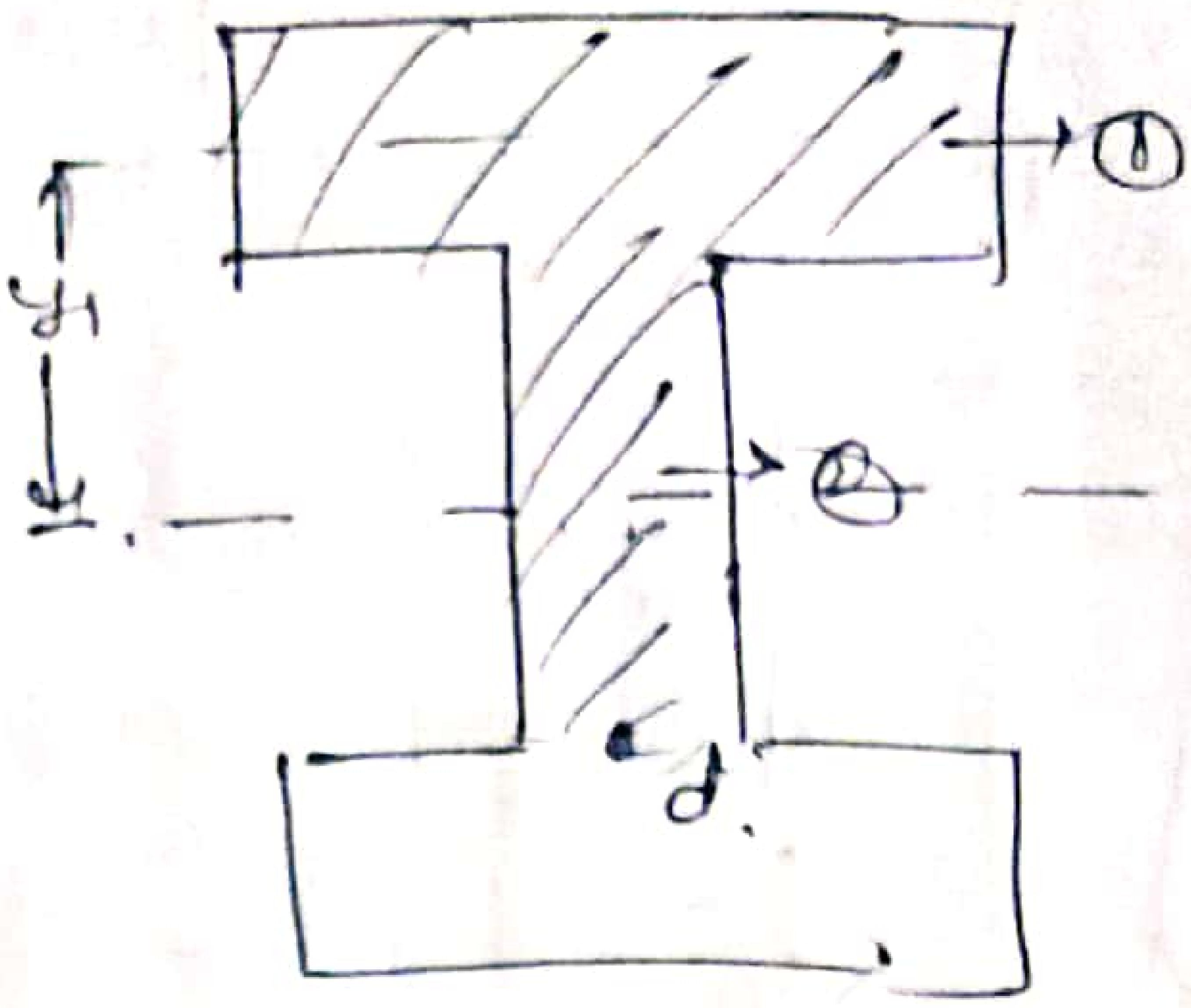
$$\tau_D = \frac{VQ_d}{It}$$



$Q_d = y_1 A_1 + y_2 A_2$. ($y_2 \to 0$)

$t = 6.1$ mm.

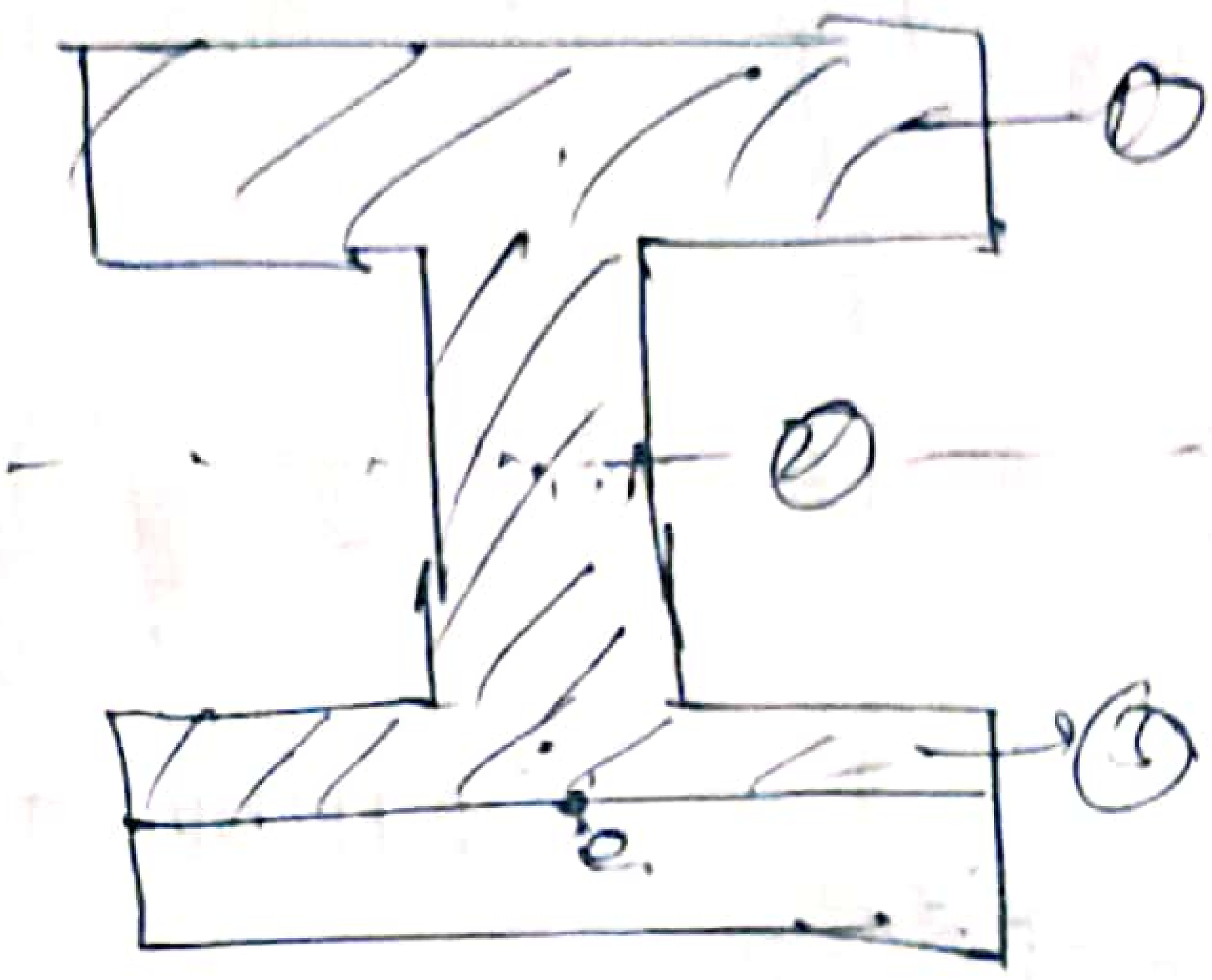
$y_1 = 124.45$

$A_1 = 9.1 \times 146$.

$$\tau_D = \frac{16 \times 10^3 \times 124.45 \times 9.1 \times 146}{49.1 \times 10^6 \times 6.1}$$

$$= 8.83 \ N/mm^2$$

$$\tau_e = \frac{VQ_e}{It}$$



$Q_e = yA_1 + y_2 A_2 + y_3 A_3$ ($y_2 \to 0$)

$y_3 = 129 - \frac{9.1}{2} - \frac{9.1}{4}$

$= -122.17$ mm.

$A_3 = +146 \times \frac{9.1}{2}$.

$$\tau_e = \frac{16 \times 10^3 \left(124.45 \times 9.1 \times 146 + (-122.17) \times (+146 \times \frac{9.1}{2})\right)}{49.1 \times 10^6 \times 146}$$

$= \cancel{.1878}$ MPa, 0.1879 MPa ~~0.55 MPa~~

$= \cancel{0.55}$ MPa.

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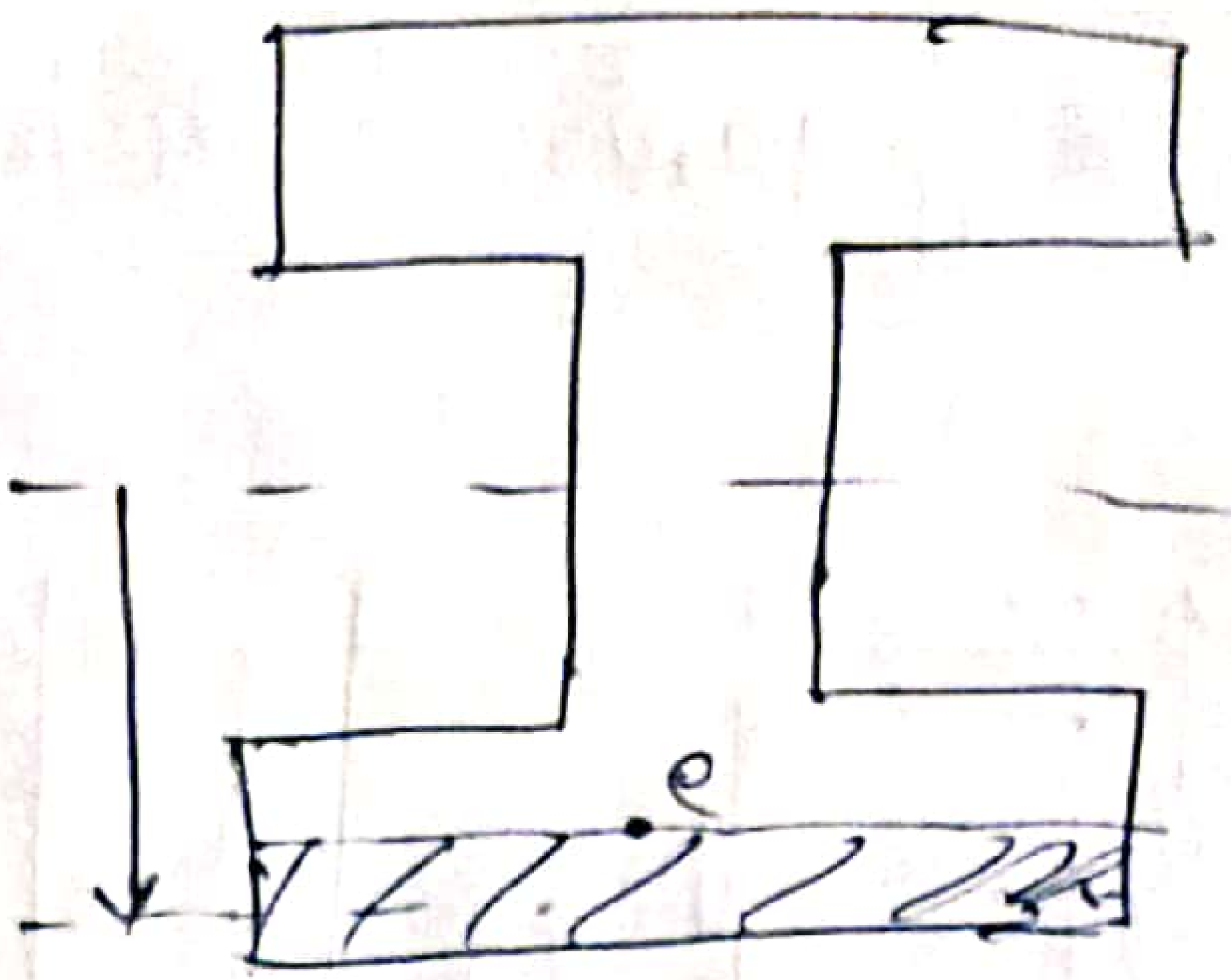
or, $\tau_e = \frac{V Q_e}{I t}$.



$Q_e = y_e A_e$

$y_e = 129.[illegible] - \frac{9.1}{4}$

$= -126.7$.

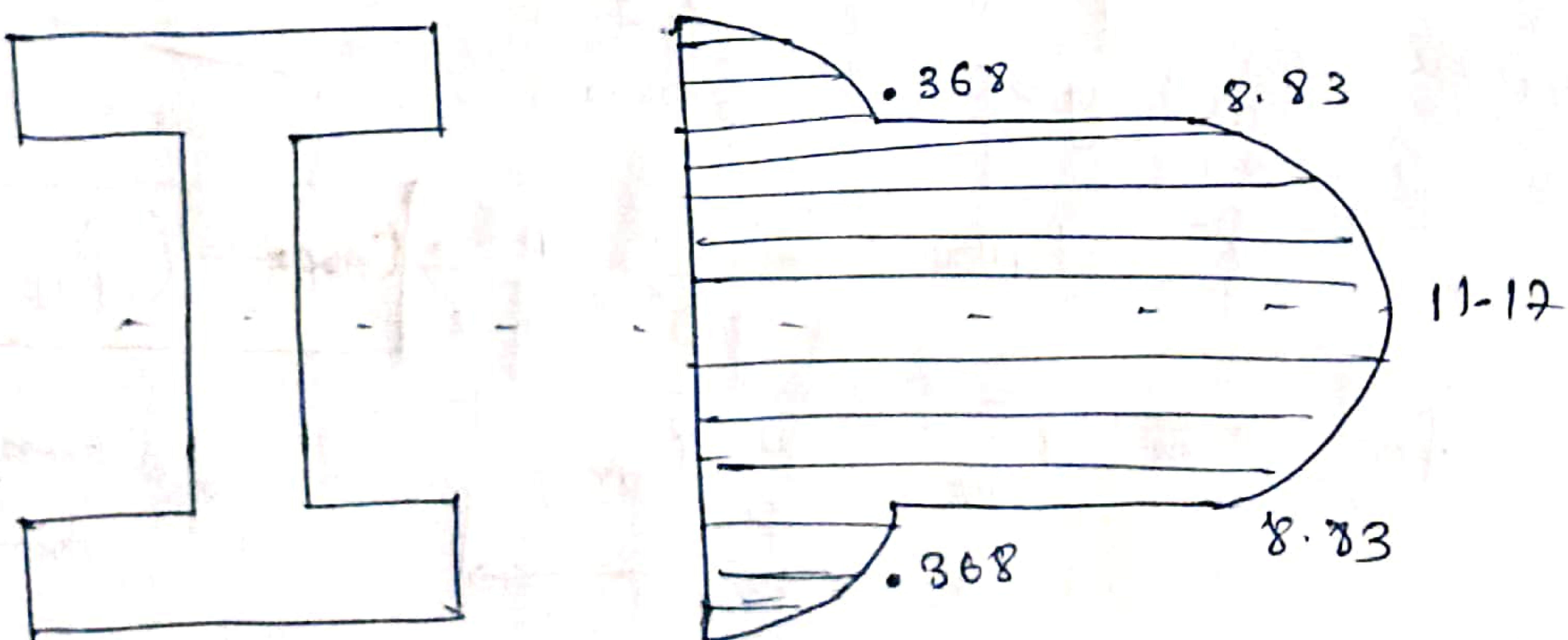
$A_e = -\frac{9.1}{2} \times 146 = -9.1 \times 73$

So $\tau = \frac{16 \times 10^3 \times 126.7 \times 9.1 \times 73}{49.1 \times 10^6 \times 146} = 0.1878$ MPa.

$\tau = \frac{VQ}{It}$ or $\tau \propto \frac{1}{t}$ or $\tau_1 t_1 = \tau_2 t_2$.

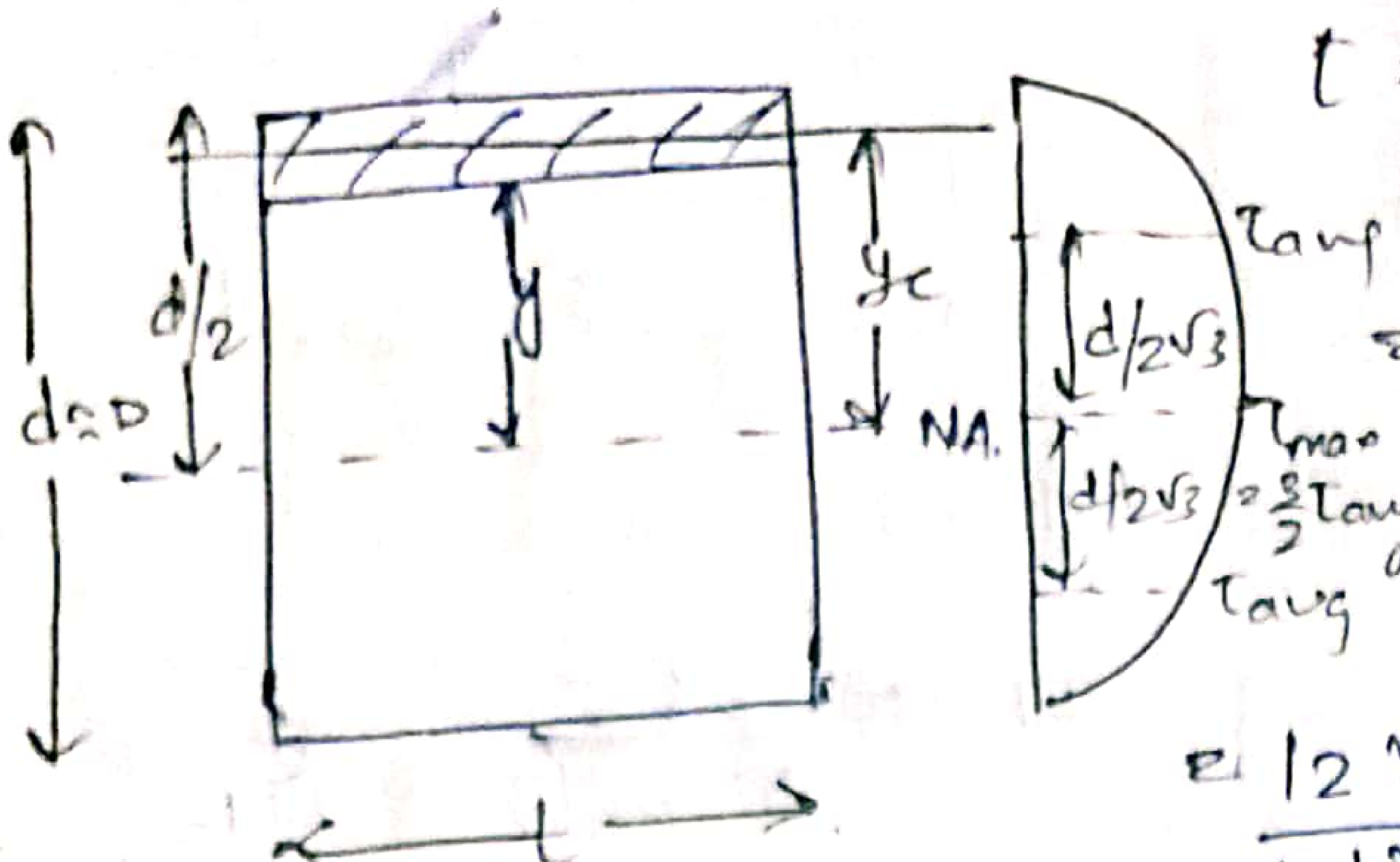
$\tau_{b^+} t_{b^+} = \tau_{b^-} t_{b^-}$

or $\tau_{b^+} = \frac{8.83 \times 6.1}{146} = .368$ MPa.



shear stress distribution (Parabolic).

Q. Why parabolic distribution in direct shear stress.



$$\tau = \frac{VQ}{It}$$

$$= \frac{V}{\left(\frac{td^3}{12}\right)t}\left[\overbrace{\left(\frac{D}{2}-y\right)t}^{A}\cdot\overbrace{\left\{\frac{D}{2}-\frac{\left(\frac{D}{2}-y\right)}{2}\right\}}^{y_c}\right]$$

$$= \frac{12V}{td^3t}\left[\frac{1}{2}\left(\frac{D}{2}-y\right)t\cdot\left(\frac{D}{2}+y\right)\right]$$

Parabolic Distribution $\boxed{\tau = \frac{6V}{tD^3}\left[\frac{D^2}{4}-y^2\right]}$

Case-(1) at $y = \pm D/2$, $\tau = 0$.

Case-(2) for τ_{max}, $\frac{d\tau}{dy} = 0$

$$\Rightarrow \frac{6V}{tD^3}\left[-2y\right] = 0 \Rightarrow y = 0$$

at $y = 0$, $\tau = \tau_{max}$.

where $V = \frac{dM}{dx}$

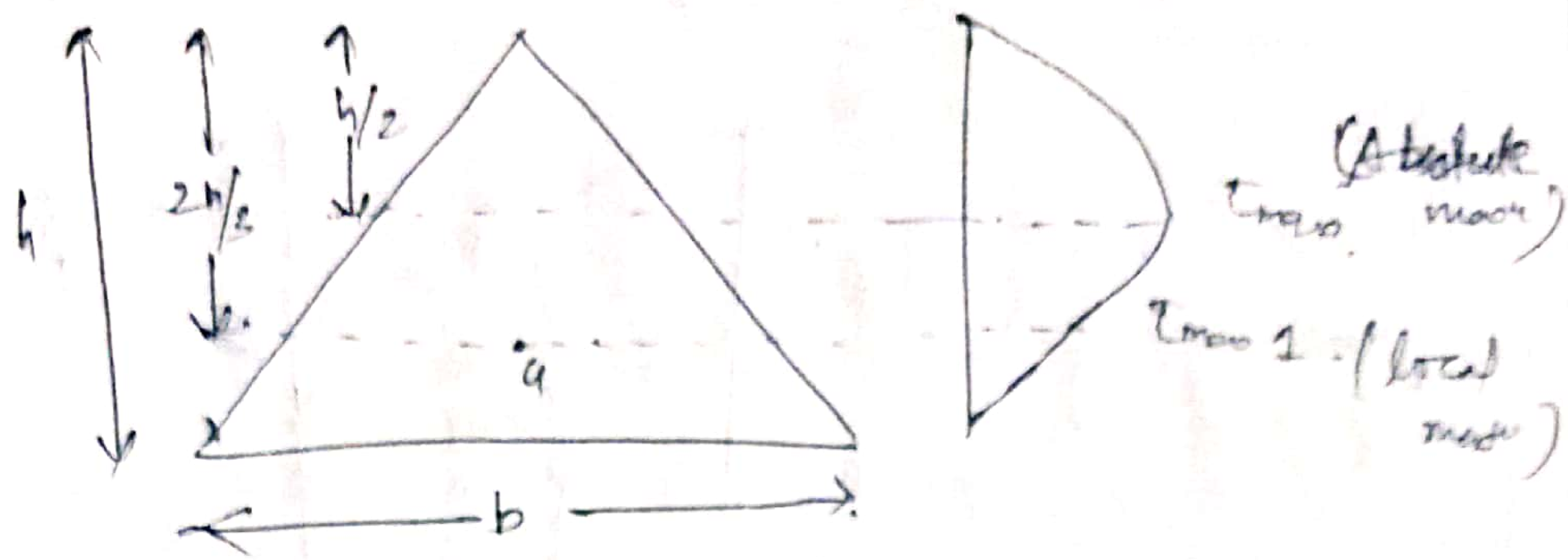
So $\tau_{max} = \frac{\cancel{6}^{3}V}{tD^{\cancel{3}}} \times \frac{D^{\cancel{2}}}{\cancel{4}_{2}} \Rightarrow \boxed{\tau_{max} = \frac{3}{2}\left(\frac{V}{tD}\right)}$

for rectangular c/s. $\boxed{\tau_{max} = \frac{3}{2}\tau_{avg}}$ & $\tau_{avg} = \frac{\text{shear force}}{\text{Area of c/s.}}$

$$\frac{6V}{tD^3}\left[\frac{D^2}{4}-y^2\right] = \frac{V}{tD}$$

$\Rightarrow \boxed{y = \pm\frac{D}{2\sqrt{3}}}$ Point of avg. shear stress.

Triangular section beam

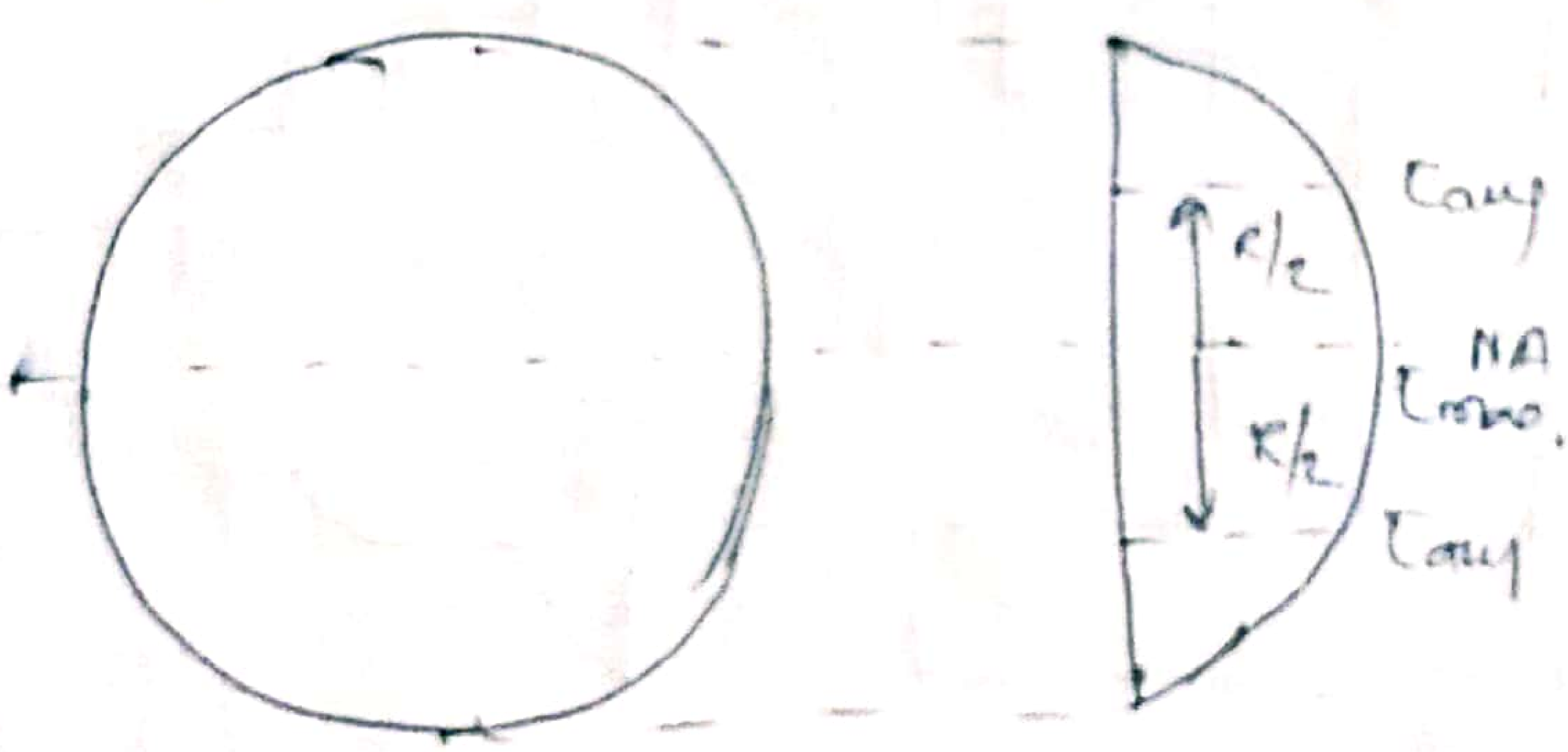


$$\tau_{avg} = \frac{V}{\frac{1}{2}bh} \quad \left(\frac{S.F}{Area}\right)$$

$$\boxed{\tau_{max} = \frac{3}{2}\tau_{avg}}$$

$$\boxed{\tau_{max\,1} = \frac{4}{3}\tau_{avg}}$$

Circular Section

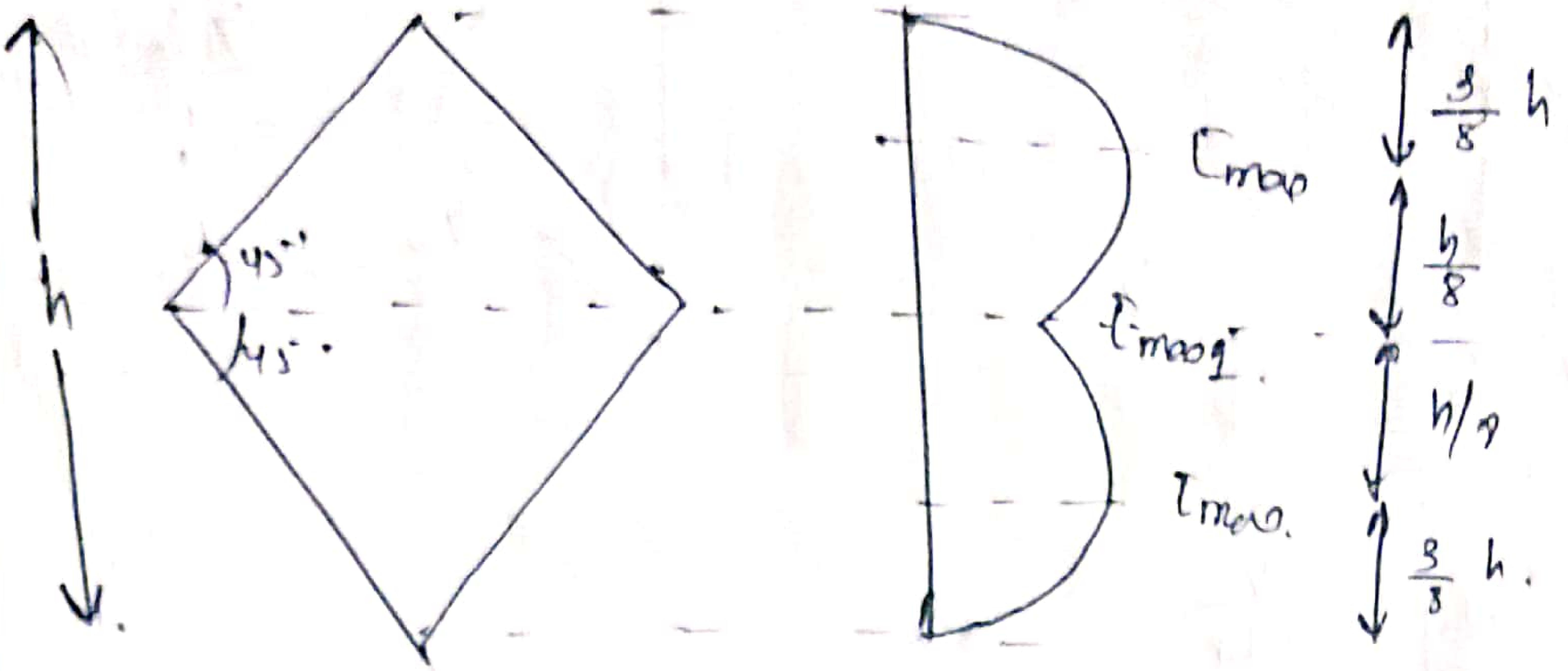


$$\tau_{avg} = \frac{V}{\left(\frac{\pi}{4}D^2\right)}$$

$\tau_{avg} \to$ at $\pm R/2$

$$\boxed{\tau_{max} = \frac{4}{3}\tau_{avg}}$$

Diamond section



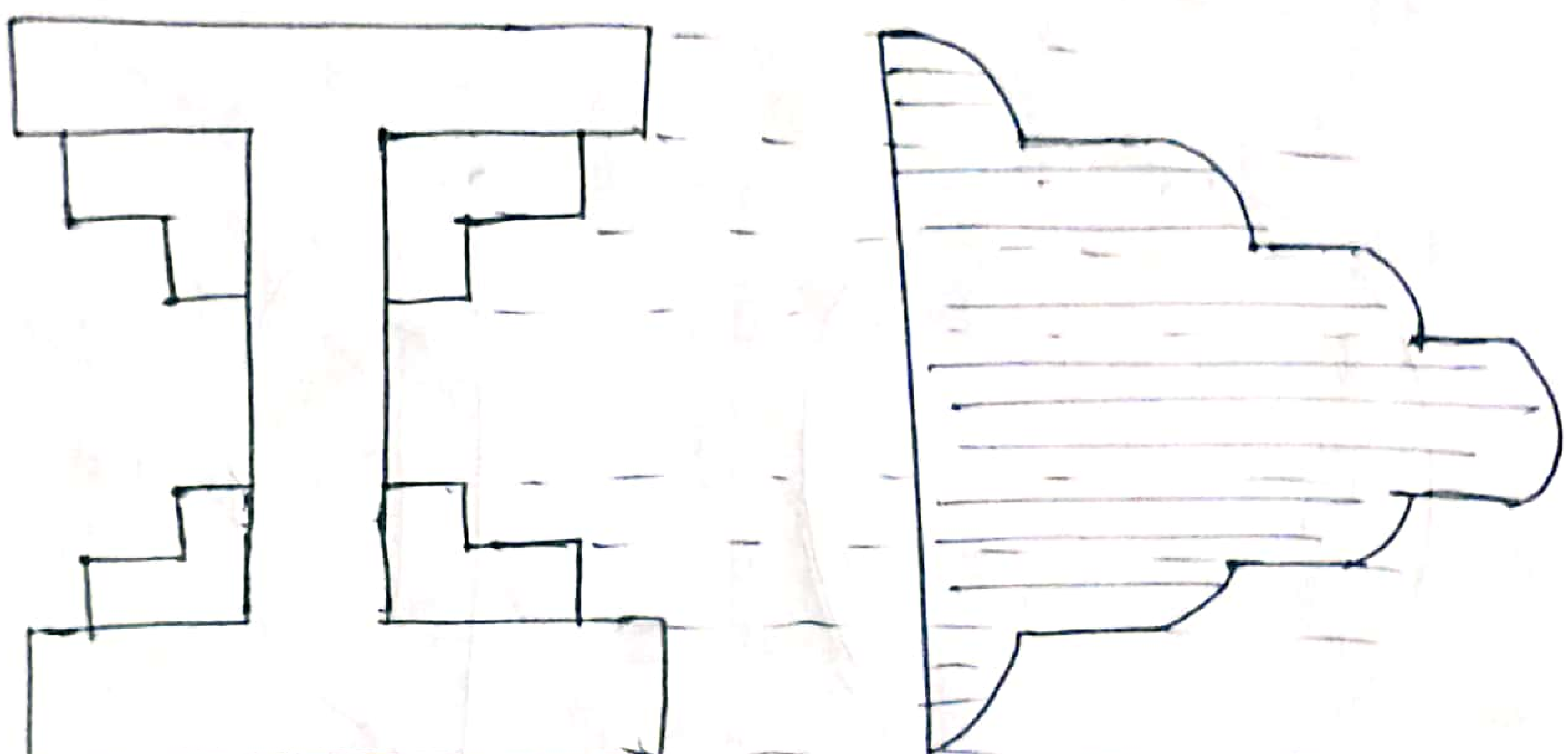
$$\boxed{\tau_{max} = \frac{9}{8}\tau_{avg}}$$

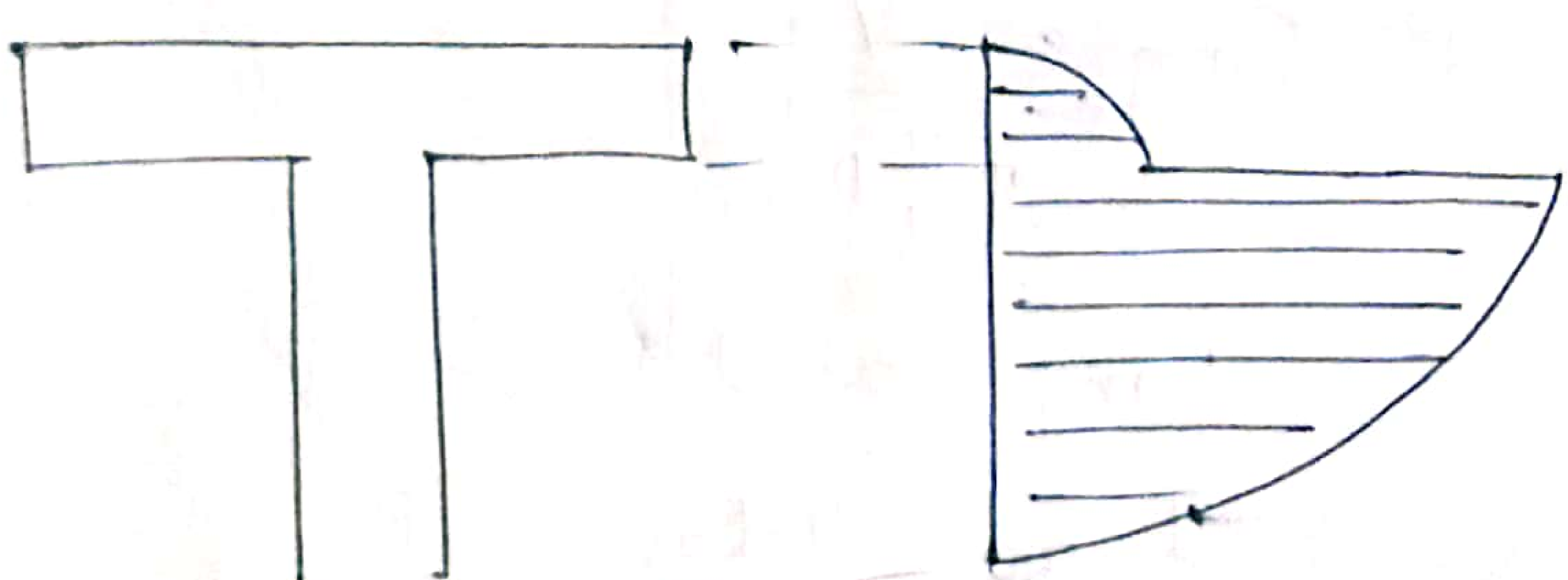
$$\boxed{\tau_{avg} = \frac{V}{2 \times \frac{1}{2} \times h \times \frac{h}{2}}}$$

Same for [illegible].

$\tau \propto \frac{1}{t}$

shear stress distribution





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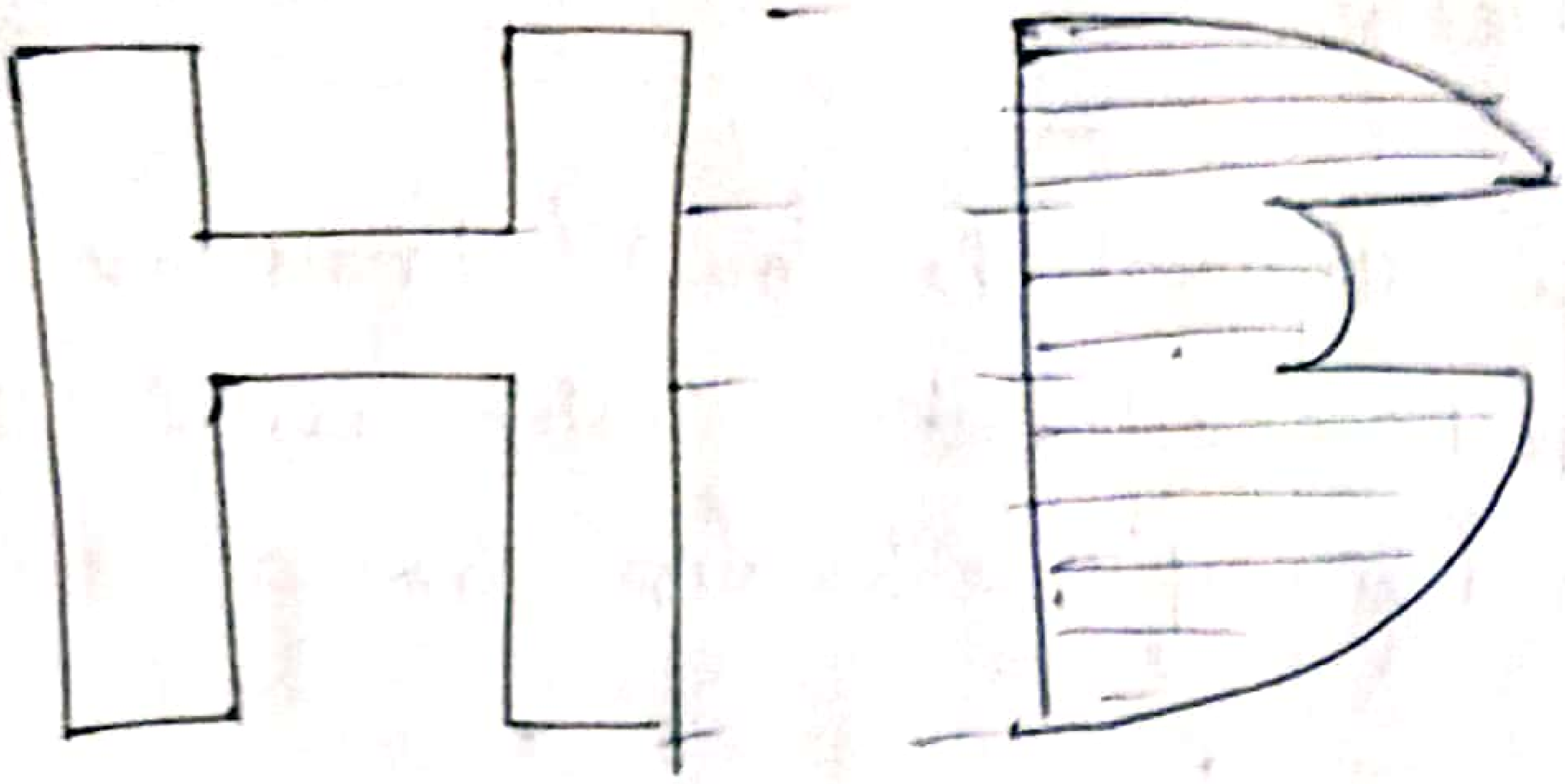

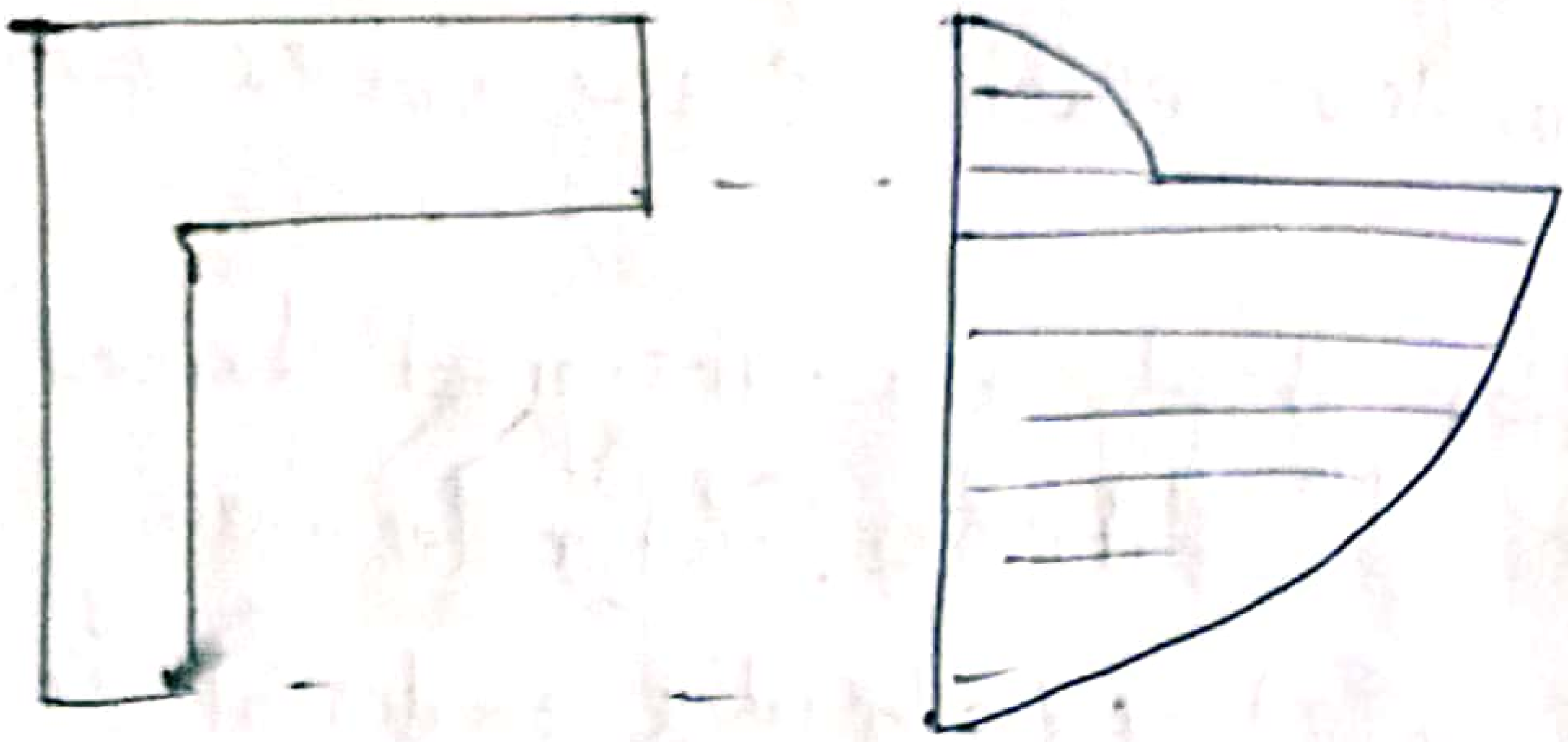

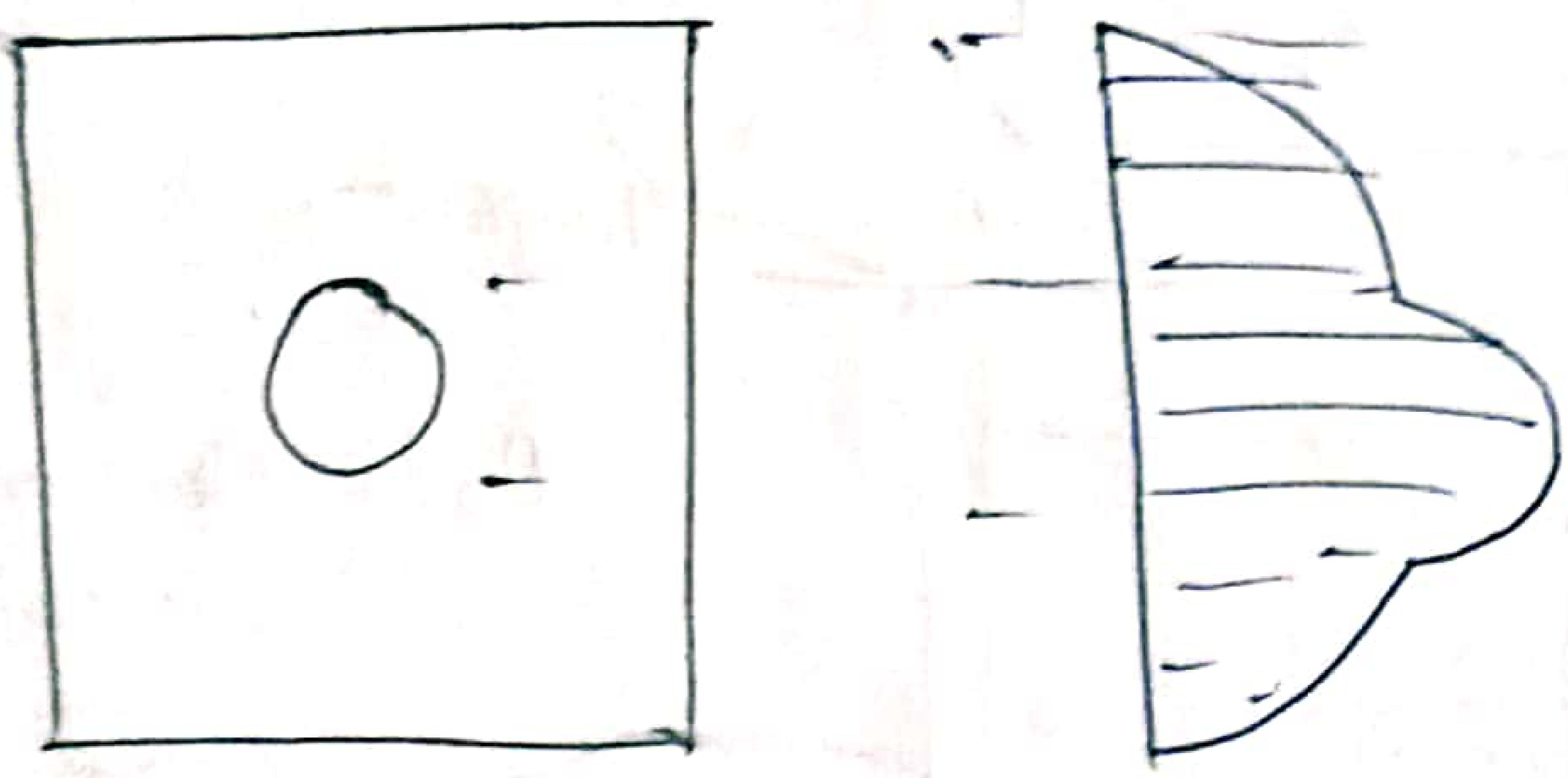

* shear flow at any section is given by = $\frac{VQ}{I}$

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